Lecture 6: Top-Down Parsing

Beating Grammars into Programs

- A BNF grammar looks like a recursive program. Sometimes it works to treat it that way.
- Assume the existence of
  - A function 'next' that returns the syntactic category of the next token (without side-effects);
  - A function 'scan(C)' that checks that the next syntactic category is C and then reads another token into next(). Returns the previous value of next().
  - A function ERROR for reporting errors.
- Strategy: Translate each nonterminal, A, into a function that reads an A according to one of its productions and returns the semantic value computed by the corresponding action.
- Result is a *recursive-descent* parser.

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp '⊣'
sexp ::= atom
       | '(' elist ')'
       | '\'' sexp
elist ::= ε
        | sexp elist
atom ::= SYM
       | NUM
       | STRING
```

```
def prog ():
   _______________

def sexp ():
   if ____________________________:
      ______
   elif _____________:
      _____________________________
   else:
      __________________

def atom ():
   if ____________________________:
      ____________
   else:
      _______

def elist ():
   if ______________________________________:
      ________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp '⊣'
sexp ::= atom
       | '(' elist ')'
       | '\'' sexp
elist ::= ϵ
        | sexp elist
atom ::= SYM
       | NUM
       | STRING
```

```
def prog ():
    sexp(); scan(⊣)

def sexp ():
    if ____________________________:
        ______
    elif _____________:
        _____________________________
    else:
        __________________

def atom ():
    if ____________________________:
        ____________
    else:
        _______

def elist ():
    if ______________________________________:
        ________________
```

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```
prog ::= sexp ’⊣’
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       | ’(’ elist ’)’
       | ’\’’ sexp
elist ::= ϵ
        | sexp elist
atom ::= SYM
       | NUM
       | STRING
```

```
def prog ():
   sexp(); scan(⊣)

def sexp ():
   if next() in [SYM, NUM, STRING]:
      atom()
   elif _____________:
      ______________________________
   else:
      __________________

def atom ():
   if ____________________________:
      ____________
   else:
      _______

def elist ():
   if ______________________________________:
      ________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp ’⊣’
sexp ::= atom
      |  ’(’ elist ’)’
      |  ’\’’ sexp
elist ::= ε
       |  sexp elist
atom ::= SYM
      |  NUM
      |  STRING
```

```
def prog ():
    sexp(); scan(⊣)

def sexp ():
    if next() in [SYM, NUM, STRING]:
        atom()
    elif next() == ’(’:
        scan(’(’); elist(); scan(’)’)
    else:
        ____________________

def atom ():
    if ______________________________:
        _____________
    else:
        _______

def elist ():
    if __________________________________________:
        _________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp ’⊣’
sexp ::= atom
       | ’(’ elist ’)’
       | ’\’’ sexp
elist ::= ϵ
        | sexp elist
atom ::= SYM
       | NUM
       | STRING
```

```
def prog ():
   sexp(); scan(⊣)

def sexp ():
   if next() in [SYM, NUM, STRING]:
      atom()
   elif next() == ’(’:
      scan(’(’); elist(); scan(’)’)
   else:
      scan(’\’’); sexp()

def atom ():
   if ____________________________:
      ____________
   else:
      _______

def elist ():
   if ______________________________________:
      ________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog  ::= sexp '⊣'
sexp  ::= atom
        | '(' elist ')'
        | '\'' sexp
elist ::= ε
        | sexp elist
atom  ::= SYM
        | NUM
        | STRING
```

```
def prog ():
    sexp(); scan(⊣)

def sexp ():
    if next() in [SYM, NUM, STRING]:
        atom()
    elif next() == '(':
        scan('('); elist(); scan(')')
    else:
        scan('\''); sexp()

def atom ():
    if next() in [SYM, NUM, STRING]:
        scan(next())
    else:
        _______

def elist ():
    if ______________________________________:
        ________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp '⊣'
sexp ::= atom
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       | STRING
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def prog ():
    sexp(); scan(⊣)

def sexp ():
    if next() in [SYM, NUM, STRING]:
        atom()
    elif next() == '(':
        scan('('); elist(); scan(')')
    else:
        scan('\''); sexp()

def atom ():
    if next() in [SYM, NUM, STRING]:
        scan(next())
    else:
        ERROR()

def elist ():
    if ____________________________________:
        ________________
```

Example: Lisp Expression Recognizer

Grammar

```
prog ::= sexp ’⊣’
sexp ::= atom
       | ’(’ elist ’)’
       | ’\’’ sexp
elist ::= ϵ
        | sexp elist
atom ::= SYM
       | NUM
       | STRING
```

```
def prog ():
    sexp(); scan(⊣)

def sexp ():
    if next() in [SYM, NUM, STRING]:
        atom()
    elif next() == ’(’:
        scan(’(’); elist(); scan(’)’)
    else:
        scan(’\’’); sexp()

def atom ():
    if next() in [SYM, NUM, STRING]:
        scan(next())
    else:
        ERROR()

def elist ():
    if next() in [SYM, NUM, STRING, ’(’, "’"]:
        sexp(); elist();
```

Expression Recognizer with Actions

- Can make the nonterminal functions return semantic values.
- Assume lexer somehow supplies semantic values for tokens, if needed

```
elist ::= ϵ                          {: RESULT = emptyList; :}
       | sexp:head elist:tail  {: RESULT = cons(head, tail); :}

def elist ():
   if next() in [SYM, NUM, STRING, '(', "'"]:

       ___________________________________________
   else:
       return emptyList
```

Expression Recognizer with Actions

- Can make the nonterminal functions return semantic values.
- Assume lexer somehow supplies semantic values for tokens, if needed

```
elist ::= ϵ                        {: RESULT = emptyList; :}
        | sexp:head elist:tail     {: RESULT = cons(head, tail); :}

def elist ():
   if next() in [SYM, NUM, STRING, '(', "'"]:
       v1 = sexp(); v2 = elist(); return cons(v1,v2)
   else:
       return emptyList
```

Grammar Problems I

In a recursive-descent parser, what goes wrong here?

```
p ::= e '⊣'
e ::= t:t1                 {: RESULT = t1; :}
    | e:lft '/' t:rgt      {: RESULT = makeTree(DIV, lft, rgt); :}
    | e:lft '*' t:rgt      {: RESULT = makeTree(MULT, lft, rgt); :}
```

Grammar Problems I

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    | e:lft '*' t:rgt  {: RESULT = makeTree(MULT, lft, rgt); :}
```

If we choose the second of third alternative for `e`, we'll get an infinite recursion. If we choose the first, we'll miss '/' and '∗' cases.

Grammar Problems II

Well then: What goes wrong here?

```
p ::= e '⊣'
e ::= t:t1               {: RESULT = t1; :}
    | t:lft '/' e:rgt {: RESULT = makeTree(DIV, lft, rgt); :}
    | t:lft '*' e:rgt {: RESULT = makeTree(MULT, lft, rgt); :}
```

Grammar Problems II

Well then: What goes wrong here?

```
p ::= e '⊣'
e ::= t:t1                {: RESULT = t1; :}
    | t:lft '/' e:rgt {: RESULT = makeTree(DIV, lft, rgt); :}
    | t:lft '*' e:rgt {: RESULT = makeTree(MULT, lft, rgt); :}
```

No infinite recursion, but we still don't know which right-hand side to choose for `e`.

FIRST and FOLLOW

- If α is any string of terminals and nonterminals (like the right side of a production) then FIRST(α) is the set of terminal symbols that start some string that α produces, plus ϵ if α can produce the empty string. For example:

  ```
  p ::= e '⊣'
  e ::= s t
  s ::= ϵ | '+' | '-'
  t ::= ID | '(' e ')'
  ```

  Since `e` $\Rightarrow$ `s t` $\Rightarrow$ `( e )` $\Rightarrow$ `...`, we know that '(' $\in$ FIRST(e). Since **s** $\Rightarrow$ ϵ, we know that $\epsilon \in$ FIRST(s).

- If X is a non-terminal symbol in some grammar, G, then FOLLOW(X) is the set of terminal symbols that can come immediately after X in some sentential form that G can produce. For example, since `p` $\Rightarrow$ `e` $\dashv$ $\Rightarrow$ `s t` $\dashv$ $\Rightarrow$ `s '(' e ')'` $\dashv$ $\Rightarrow$ `...`, we know that '(' $\in$ FOLLOW(s).

Using FIRST and FOLLOW

- In a recursive-descent compiler where we have a choice of right-hand sides to produce for non-terminal, X, look at the FIRST of each choice and take it if the next input symbol is in it...
- ...and if a right-hand side's FIRST set contains ϵ, take it if the next input symbol is in FOLLOW(X).

Grammar Problems III

What actions?

```
p   ::= e '⊣'
e   ::= t et          {: ?1 :}
et  ::= ϵ             {: ?2 :}
     | '/' e          {: ?3 :}
     | '*' e          {: ?4 :}
t   ::= I:i1          {: RESULT = i1; :}
```

What are FIRST and FOLLOW?

Grammar Problems III

What actions?

```
p   ::= e ’⊣’
e   ::= t et          {: ?1 :}
et  ::= ϵ             {: ?2 :}
      | ’/’ e         {: ?3 :}
      | ’*’ e         {: ?4 :}
t   ::= I:i1          {: RESULT = i1; :}
```

Here, we don’t have the previous problems, but how do we build a tree that associates properly (left to right), so that we don’t interpret `I/I/I` as if it were `I/(I/I)`?

What are FIRST and FOLLOW?

Grammar Problems III

What actions?

```
p   ::= e '⊣'
e   ::= t et          {: ?1 :}
et  ::= ϵ             {: ?2 :}
     |  '/' e         {: ?3 :}
     |  '*' e         {: ?4 :}
t   ::= I:i1          {: RESULT = i1; :}
```

Here, we don't have the previous problems, but how do we build a tree that associates properly (left to right), so that we don't interpret `I/I/I` as if it were `I/(I/I)`?

What are FIRST and FOLLOW?

```
FIRST(p) = FIRST(e) = FIRST(t) = { I }
FIRST(et) = { ϵ, '/', '*' }
FIRST('/' e) = { '/' }      (when to use ?3)
FIRST('*' e) = { '*' }      (when to use ?4)
FOLLOW(e) = { '⊣' }
FOLLOW(et) = FOLLOW(e)      (when to use ?2)
FOLLOW(t) = { '⊣', '/', '*' }
```

Using Loops to Roll Up Recursion

- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    _______
    while ____________________:
        if _____________:
            ___________________
            _________________________
        else:
            ___________________
            __________________________
    return _
```

Using Loops to Roll Up Recursion

- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    r = t()
    while ____________________:
        if _____________:
            ___________________
            _________________________
        else:
            ___________________
            __________________________
    return _
```

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- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    r = t()
    while next() in ['/', '*']:
        if ____________:
            ___________________
            _________________________
        else:
            ___________________
            __________________________
    return _
```

Using Loops to Roll Up Recursion

- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    r = t()
    while next() in ['/', '*']:
       if next() == '/':
           scan('/'); t1 = t()
           r = makeTree (DIV, r, t1)
       else:
           ___________________
           __________________________
    return _
```

Using Loops to Roll Up Recursion

- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    r = t()
    while next() in [’/’, ’*’]:
        if next() == ’/’:
            scan(’/’); t1 = t()
            r = makeTree (DIV, r, t1)
        else:
            scan(’*’); t1 = t()
            r = makeTree (MULT, r, t1)
    return _
```

Using Loops to Roll Up Recursion

- There are ways to deal with problem in last slide within the pure framework, but why bother?
- Implement e procedure with a loop, instead:

```
def e():
    r = t()
    while next() in ['/', '*']:
        if next() == '/':
            scan('/'); t1 = t()
            r = makeTree (DIV, r, t1)
        else:
            scan('*'); t1 = t()
            r = makeTree (MULT, r, t1)
    return r
```

From Recursive Descent to Table Driven

- Our recursive descent parsers have a very regular structure.

Definition of nonterminal A:

```
A ::= $\alpha_1$
    | $\alpha_2$
    | ...
    | $\alpha_3$
```

Program for A:

```
def A():
    if next() in $S_1$:
        translation of $\alpha_1$
    elif next() in $S_2$:
        translation of $\alpha_2$
    ...
```

- Here,

$$S_i = \left\{ \begin{array}{ll} \text{FIRST}(\alpha_i), & \text{if } \epsilon \notin \text{FIRST}(\alpha_i) \\ \text{FIRST}(\alpha_i) \cup \text{FOLLOW}(A), & \text{otherwise.} \end{array} \right\}$$

- and the translation of α_i simply converts each nonterminal into a call and each terminal into a `scan`.
- If the S_i do not overlap, we say the grammar is LL(1): input can be processed from Left to right, producing a Leftmost derivation, and checking 1 symbol of input ahead to see which branch to take.

Table-Driven LL(1)

- Because of this regular structure, we can represent the program as a table, and can write a general LL(1) parser that interprets any such table.
- Consider a previous example:

Grammar

```
1. prog  ::= sexp '⊣'
2. sexp  ::= atom
3.         | '(' elist ')'
4.         | '\'' sexp
5. elist ::= ε
6.         | sexp elist
7. atom  ::= SYM
8.         | NUM
9.         | STRING
```

Nonterminal	Lookahead symbol						
	(	)	'	SYM	NUM	STRING	⊣
prog	(1)		(1)	(1)	(1)	(1)	
sexp	(3)		(4)	(2)	(2)	(2)	
elist	(6)	(5)	(6)	(6)	(6)	(6)	(5)
atom				(7)	(8)	(9)	

- The table shows nonterminal symbols in the left column and the other columns show which production to use for each possible lookahead symbol.
- Grammar is LL(1) when this table has at most one production per entry.

A General LL(1) Algorithm

Given a fixed table T and grammar G, the function LLparse(X), where parameter X is a grammar symbol, may be defined

```
def LLparse(X):
    if X is a terminal symbol:
        scan(X)
    else:
        prod = T[X][next()]
        Let $p_1 p_2 \cdots p_n$ be the right-hand side of production prod
        for i in range(n):
            LLparse($p_i$)
```