Bit-serial Adder

- A, B, and R held in shift-registers. Shift right once per clock cycle.
- Reset is asserted by controller.
- Addition of 2 n-bit numbers:
  - takes n clock cycles.
  - uses 1 FF, 1 FA cell, plus registers
  - the bit streams may come from or go to other circuits, therefore the registers are optional.

Multiplication

\[
\begin{align*}
X & = a_3b_3 + a_2b_2 + a_1b_1 + a_0b_0 \\
\text{Partial products} & = a_3b_3 \quad a_2b_2 \quad a_1b_1 \quad a_0b_0 \\
\ldots & = a_n b_n
\end{align*}
\]

Many different circuits exist for multiplication. Each one has a different balance between speed (performance) and amount of logic (cost).

8-bit Carry Look-ahead Adder

- P, G inputs to X, Y output:
  - Xilinx 4000XL series:
  - Carry Net Delay, \( C_{\text{IN}} \) to \( C_{\text{OUT}} \):
    - 1.1ns
  - Ripple adders of 16 or even 32 bits sufficiently low delay for most applications.

- If high-speed or wider adders are needed ripple adders can be expanded using carry select or carry look-ahead technique.

“Shift and Add” Multiplier

- Sums each partial product, one at a time.
- In binary, each partial product is shifted versions of A or 0.

Control Algorithm:
1. \( P \leftarrow 0 \), \( A \leftarrow \) multiplicand.
2. If LSB of \( B \) is 1 then add \( A \) to \( P \) else add 0
3. Shift \( P, B \) right 1
4. Repeat steps 2 and 3 \( n-1 \) times.
5. \( [P,B] \) has product.

Xilinx Carry-chain

- Most FPGAs include special high-speed carry logic to speed up ripple adders:
  - Xilinx 4000XL series:
  - Ripple adders of 16 or even 32 bits sufficiently low delay for most applications.

- If high-speed or wider adders are needed ripple adders can be expanded using carry select or carry look-ahead technique.
Signed Multiplication:
Remember for 2's complement numbers MSB has negative weight:
\[ X = \sum x_i 2^i \]
\[ 11010_2 = 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 - 1 \cdot 2^0 = 0 + 0 + 8 + 0 - 16 = -6 \]
• Therefore for multiplication:
  a) subtract final partial product
  b) sign-extend partial products
• Modifications to shift & add circuit:
  a) adder/subtractor
  b) sign-extender on P shifter register