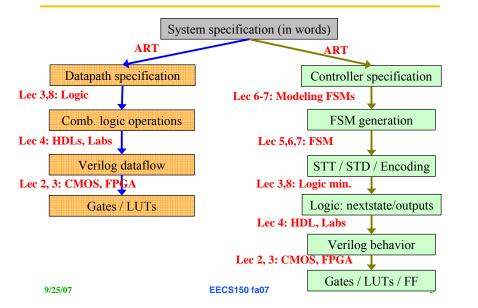


### Digital design - as we've seen it



### Where are we now?

- (Synchronous) Sequential systems
- Given datapath and control specifications
  - Generate comb. logic for datapath
    - » Minimize logic for efficient implementation
  - Generate FSM for controller
    - » Choose implementation, encoding
    - » Generate logic for nextstate and output
  - Describe datapath and controller in Verilog
    - » structure, dataflow and behavior
    - » Map onto gates or LUTs
- Seems like a good point to "test" your understanding!

### **Representation of digital designs**

scope of CS 150

more depth than 61C

focus on building systems

5

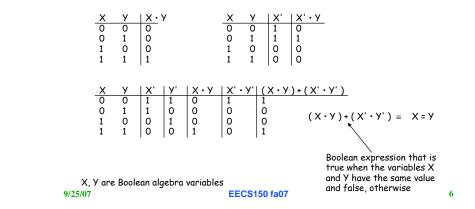
- Physical devices (transistors, relays)
- Switches
- Truth tables
- Boolean algebra
- Gates

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- Waveforms
- · Finite state behavior
- Register-transfer behavior
- Concurrent abstract specifications

### Logic Functions and Boolean Algebra

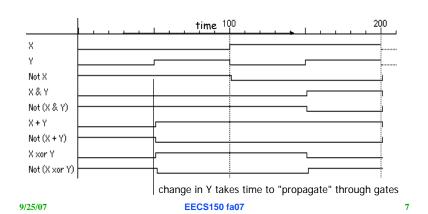
 Any logic function that can be expressed as a truth table can be written as an expression in Boolean algebra using the operators: ', +, and •



### **Waveform View of Logic Functions**

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- Just a sideways truth table
  - But note how edges don't line up exactly
  - It takes time for a gate to switch its output!



### An algebraic structure

- · An algebraic structure consists of
  - a set of elements B
  - binary operations { + , }
  - and a unary operation { ' }
  - such that the following axioms hold:

1. set B contains a	at least two elements, a, b, s	such that a ≠ b
2. closure:	a+b is in B	a∙b isinB
3. commutativity:	a + b = b + a	a • b = b • a
4. associativity:	a + (b + c) = (a + b) + c	a • (b • c) = (a • b) • c
5. identity:	a + 0 = a	a • 1 = a
6. distributivity:	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
7. complementarit	ty: a + a' = 1	a • a' = 0

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### **Timing Methodologies (cont'd)**

- Definition of terms
  - clock: periodic event, causes state of storage element to change; can be rising or falling edge, or high or low level
  - setup time: minimum time before the clocking event by which the input must be stable (Tsu)
  - hold time: minimum time after the clocking event until which the input must remain stable (Th)

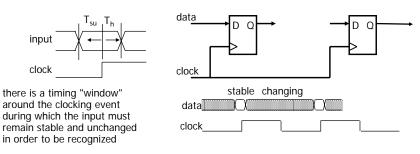


there is a timing "window" around the clocking event

in order to be recognized

clock

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### **Axioms & theorems of Boolean algebra**

Identity	
1. $X + 0 = X$	1D. $X \cdot 1 = X$
• Null	
2. X + 1 = 1	2D. $X \cdot 0 = 0$
<ul> <li>Idempotency:</li> </ul>	
3. X + X = X	3D. X • X = X
<ul> <li>Involution:</li> </ul>	
4. (X')' = X	
<ul> <li>Complementarity:</li> </ul>	
5. X + X' = 1	5D. X • X' = 0
Commutativity:	
6. $X + Y = Y + X$	$6D.  X \bullet Y = Y \bullet X$
<ul> <li>Associativity:</li> </ul>	
7. $(X + Y) + Z = X + (Y + Z)$	7D. $(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$
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### Axioms and theorems of Boolean algebra (cont'd)

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• Distributivity: 8.  $X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$  8D.  $X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$ • Uniting: 9.  $X \cdot Y + X \cdot Y' = X$ 9D.  $(X + Y) \cdot (X + Y') = X$ • Absorption: 10.  $X + X \cdot Y = X$ 10D.  $X \cdot (X + Y) = X$ 11.  $(X + Y') \cdot Y = X \cdot Y$ 11D.  $(X \bullet Y') + Y = X + Y$ • Factoring: 12.  $(X + Y) \cdot (X' + Z) =$ 12D.  $X \cdot Y + X' \cdot Z =$  $(X + Z) \cdot (X' + Y)$  $X \bullet Z + X' \bullet Y$ • Concensus: 13.  $(X \bullet Y) + (Y \bullet Z) + (X' \bullet Z) = 13D. (X + Y) \bullet (Y + Z) \bullet (X' + Z) =$  $(X + Y) \cdot (X' + Z)$  $X \bullet Y + X' \bullet Z$ 

### Axioms and theorems of Boolean algebra (cont')

14D.  $(X \bullet Y \bullet ...)' = X' + Y' + ...$ 

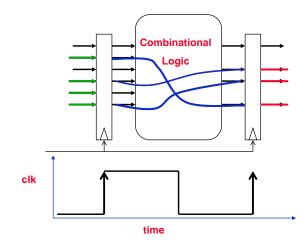
• de Morgan's:

14.  $(X + Y + ...)' = X' \cdot Y' \cdot ...$ 

- generalized de Morgan's: 15.  $f'(X1, X2, ..., Xn, 0, 1, +, \bullet) = f(X1', X2', ..., Xn', 1, 0, \bullet, +)$
- establishes relationship between and +



### **Recall: What makes Digital Systems tick?**



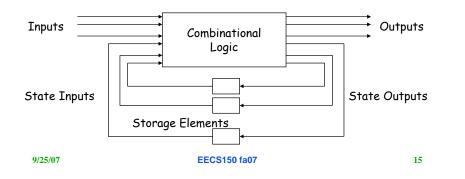
### **Sequential Logic Implementation**

# Models for representing sequential circuits Finite-state machines (Moore and Mealy) Representation of memory (states) Changes in state (transitions) Design procedure State diagrams Implementation choice: counters, shift registers, FSM State transition table State encoding Combinational logic Next state functions Output functions



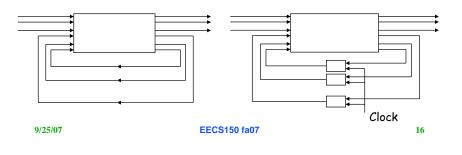
### **Abstraction of State Elements**

- Divide circuit into combinational logic and state
- · Localize feedback loops and make it easy to break cycles
- Implementation of storage elements leads to various forms of sequential logic



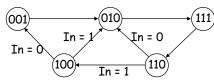
### Forms of Sequential Logic

- Asynchronous sequential logic state changes occur whenever state inputs change (elements may be simple wires or delay elements)
- Synchronous sequential logic state changes occur in lock step across all storage elements (using a periodic waveform - the clock)



### **FSM** Representations

- States: determined by possible values in sequential storage elements
- Transitions: change of state
- Clock: controls when state can change by controlling storage elements

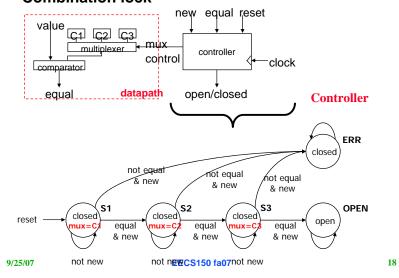


- Sequential Logic
  - Sequences through a series of states
  - Based on sequence of values on input signals
  - Clock period defines elements of sequence

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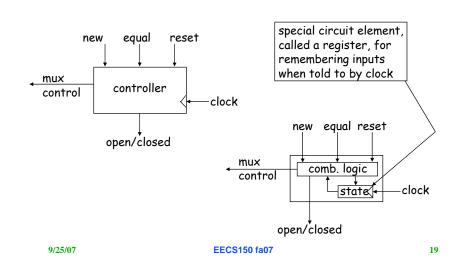
### Example: FSM Design – Combo lock

### Combination lock

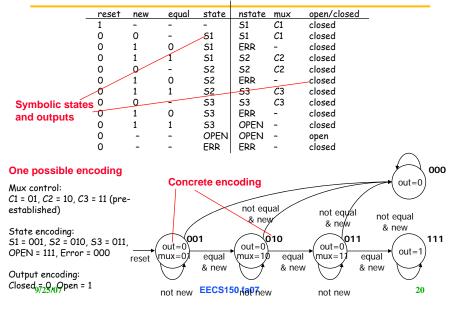


# Combo lock - controller implementation

• Implementation of the controller



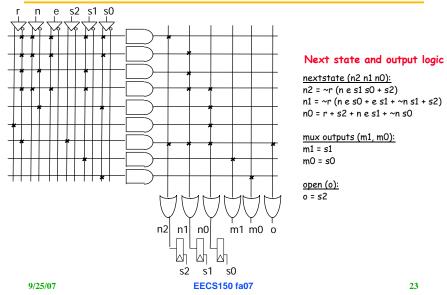
### **Combo Lock - State Encoding**



### **FSM** implementation

Steps for the hardware designer:		reset	new	equal	state	nstate	mux	open	
<ul> <li>Word specification</li> </ul>		(r)	(n)	(e)	(s <sub>2</sub> s <sub>1</sub> s <sub>0</sub> )	(n <sub>2</sub> n <sub>1</sub> n <sub>0</sub> )	(m <sub>1</sub> m <sub>0</sub> )	(0)	Next state and output log
<ul> <li>FSM design</li> </ul>		1	-	-		001		0	nextstate (n2 n1 n0):
<ul> <li>Encoding</li> </ul>		0	0	-	001	001	01	0	n2 = ~r (n e s1 s0 + s2)
– Verification!		0	1	0	001	000	01	0	n1 = -r (n e s0 + e s1 + -n s1 + s)
		0	1	1	001	010	01	0	n0 = r + s2 + n e s1 + ~n s0
At this point, hand over to synthesis tools:		0	0	-	010	010	10	0	<u>mux outputs (m1, m0):</u>
<ul> <li>Describe FSM behavior in Verilog</li> </ul>		0	1	0	010	000	10	0	m1 = s1
<ul> <li>Synthesize controller</li> </ul>		0	1	1	010	011	10	0	mO = sO
		0	0	-	011	011	11	0	<u>open (o):</u> o = s2
Good encoding		0	1	0	011	000	11	0	
<ul> <li>Better performance</li> </ul>		0	1	1	011	111	11	0	Take advantage of DC:
- Fewer state bits		0	-	-	111	111		1	
<ul> <li>Possibility of state minimization</li> </ul>		0	-	-	000	000		0	
<ul> <li>Tools also try to figure this out</li> </ul>		0	-	-	100			-	How do we get these:
		0	-	-	101			-	•K-maps?
for this example, so through the logic synthesis store		0	-	-	110			-	•Tools ≻Espresso
For this example, go through the logic synthesis steps									>Synplicity
ideally, tools take care of all this).	21		9/25/07			EECS	S150 fa07		22 22

### Logic Implementation (on PLA)



### **Alternate logic implementations**

- PALs
- Multi-level circuits

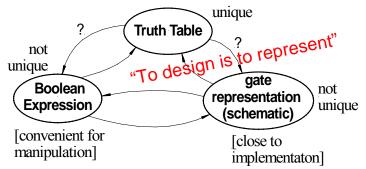
**Example: Combo Lock** 

- Library of gates for implementation technology
- LUTs on FPGA
- ...

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### **Alternate Logic Representations**

Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.



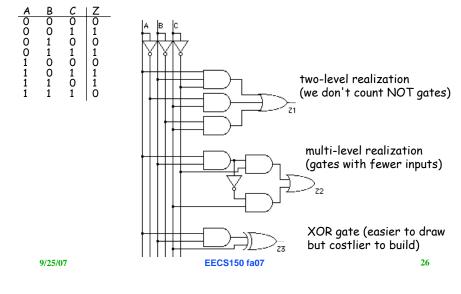
How do we convert from one to the other?

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## Which realization is best?

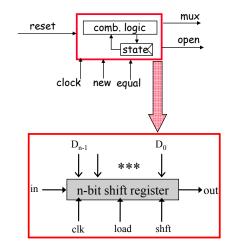
- Reduce number of inputs
  - literal: input variable (complemented or not)
    - » approximate cost of logic gate is 2 transistors per literal
  - Fewer literals means less transistors smaller circuits
  - Fewer inputs implies faster gates
  - Fan-ins (# of gate inputs) are limited in some technologies
- Reduce number of gates
  - Fewer gates (and the packages they come in) means smaller circuits
- Reduce number of levels of gates
  - Fewer level of gates implies reduced signal propagation delays
- How do we explore tradeoffs between increased circuit delay and size?
  - Automated tools to generate different solutions
  - Logic minimization: reduce number of gates and complexity
  - Logic optimization: reduction while trading off against delay

# Choosing different realizations of a function



# Alternate Implementation: Controller based on Shift Register

- Previous implementation
  - Comb. logic as gates (PLA)
  - State bits in latches
- Alternative
  - Shift reg to manipulate state
  - Simplify comb. logic

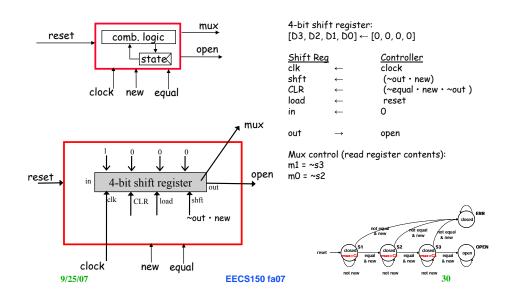


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### **Controller using Shift Register**

reset	new	equal	state	nstate	mu×	open	
(r)	(n)	(e)	$(s_3s_2s_1s_0)$	(n <sub>3</sub> n <sub>2</sub> n <sub>1</sub> n <sub>0</sub> )	(m <sub>1</sub> m <sub>0</sub> )	(o)	
1	-	-		1000		0	
0	0	-	1000	1000	01	0	One-hot encoding scheme: state
0	1	0	1000	0000	01	0	transition is a shift right
0	1	1	1000	0100	01	0	Mux control:
0	0	-	0100	0100	10	0	C1 = 01, C2 = 10, C3 = 11 (pre-
0	1	0	0100	0000	10	0	established)
0	1	1	0100	0010	10	0	
0	0	-	0010	0010	11	0	State encoding:
0	1	0	0010	0000	11	0	51 = 1000, 52 = 0100,
0	1	1	0010	0001	11	0	S3 = 0010, OPEN = 0001, Error = 0000
0	-	-	0001	0001		1	0000
0	-	-	0000	0000		0	Output encoding:
0	-	-	11			-	Closed = 0, Open = 1
0	-	-	1-1-			-	closed
0	-	-	11			-	not equal not equal since the second
0	-	-	-11-			-	
0	-	-	-1-1			-	reset
0	<u>-</u> 9/25/07	-	11		ECS150 fa07	-	not new not new not new 29

### Combo lock controller on shift reg

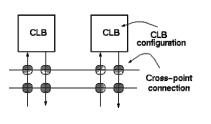


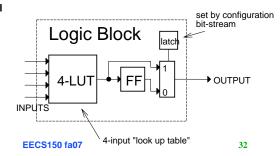
# How does the combo lock look on an FPGA?

- Latches
  - implement shift register (chain of 4 latches)
- LUTs
  - Combinational logic for out and mux control
- Routing fabric
  - Connect logical nets between CLBs

### **Inside the FPGA**

- Network of Combinational logic blocks, memory and I/O
  - rich interconnect network
  - special units multipliers, carry-logic
- CLBs
  - 3 or 4-input look up table (LUT)
  - implements combinational logic functions
  - Register optionally stores output of LUT
- Logic on FPGA
  - Configure LUTs (table of entries)
  - Configure latches in CLB
  - Program interconnect

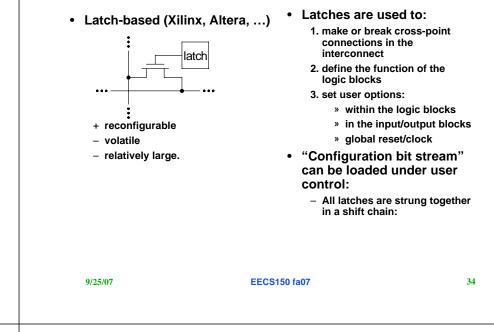




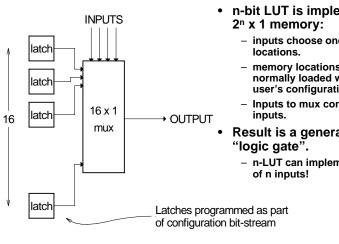
### LUT as general logic gate Example: 4-lut

		ampi <del>o</del> . <del>4</del> -iui
<ul> <li>An n-lut as a direct</li> </ul>	INPUTS	
implementation of a function	0000	F(0,0,0,0) < store in 1st latch
truth-table.	0001	F(0,0,0,1) < store in 2nd latch
<ul> <li>Each latch location holds the</li> </ul>	0010	F(0,0,1,0) <
value of the function	0011	F(0,0,1,1) <
corresponding to one input	0011	_
combination.	0100	•
Example: 2-lut	0101	•
INPUTS, AND OR	0110	
	0111	
	1000	
	1001 1010	
	1010	
	1100	
Implements any function of 2 inputs.	1101	
How many of these are there?	1110	
•	1111	
How many functions of n inputs?		
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### **User Programmability**



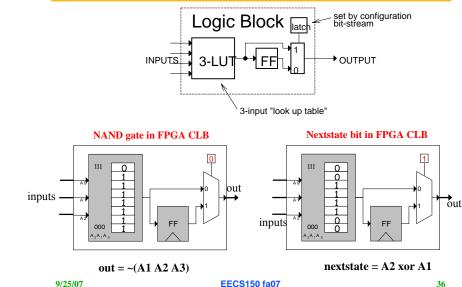
### **4-LUT Implementation**



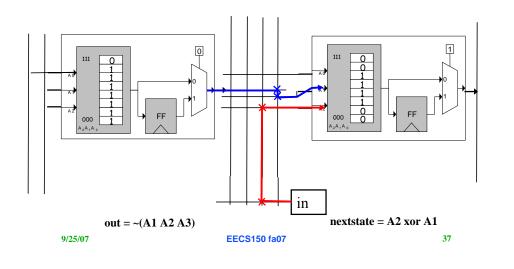
## · n-bit LUT is implemented as a

- inputs choose one of 2<sup>n</sup> memory
- memory locations (latches) are normally loaded with values from user's configuration bit stream.
- Inputs to mux control are the CLB
- Result is a general purpose
  - n-LUT can implement any function

### **Configuring CLBs**



### **Configuring Routes**



### Sequential Systems – more examples

- Beat the combo lock example to death
  - Direct FSM implementation
  - Shift register
    - » Multiple logic representations
    - » gates to LUTs

### • Up next

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- A few quick counter examples
- Another design problem Ant Brain

### Announcements/Reminders

- First mid term Thursday 9/27
  - No notes (... to discuss)
  - Review materials are in the HW4
  - Review session tonight 8-10 642-WALK (9255)
  - Trying to make the exams routine
- · Feel free to approach us with questions...
- No discussion Thurs, yes friday
- Lab 5 Where's the music?
  - Normal lab lecture on Friday

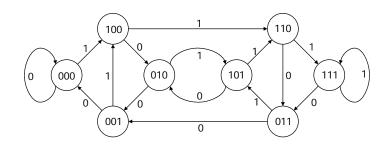
### Can Any Sequential System be Represented with a State Diagram?

• Shift Register



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Output values shown within state node



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**40** 

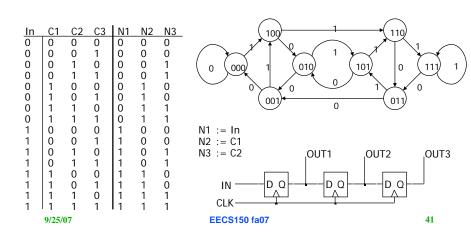
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OUT3

### **Counter Example**

### • Shift Register

Input determines next state

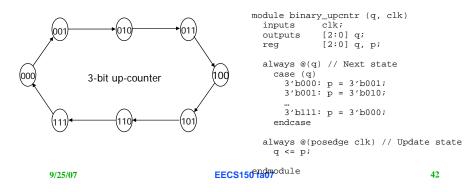


### **Counters are Simple Finite State Machines**

Counters

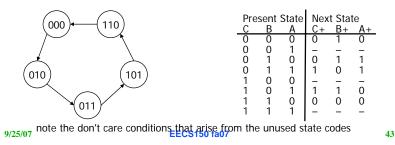
- Proceed thru well-defined state sequence in response to enable

- Many types of counters: binary, BCD, Gray-code
  - 3-bit up-counter: 000, 001, 010, 011, 100, 101, 110, 111, 000, ...
  - 3-bit down-counter: 111, 110, 101, 100, 011, 010, 001, 000, 111, ...



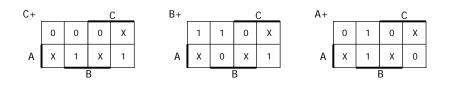
### **More Complex Counter Example**

- Complex Counter
  - Repeats five states in sequence
  - Not a binary number representation
- Step 1: Derive the state transition diagram
  - Count sequence: 000, 010, 011, 101, 110
- Step 2: Derive the state transition table from the state transition diagram



# More Complex Counter Example (cont'd)

• Step 3: K-maps for Next State Functions



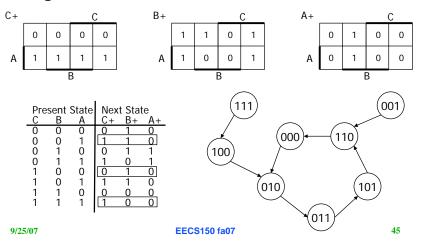


A+ := BC'

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### Self-Starting Counters (cont'd)

 Re-deriving state transition table from don't care assignment



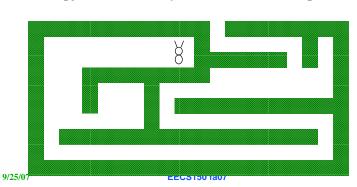
### Final Example: Ant Brain (Ward, MIT)

• Sensors: touching wall L and R antennae, 1 if in

keep the wall on the right

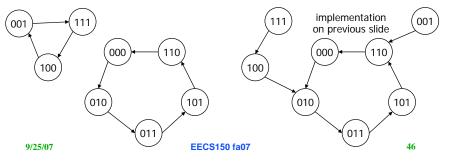
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- F forward step, TL/TR turn • Actuators: left/right slightly find way out of maze
- · Goal:
- Strategy:

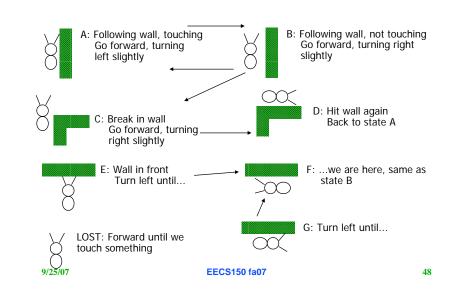


## **Self-Starting Counters**

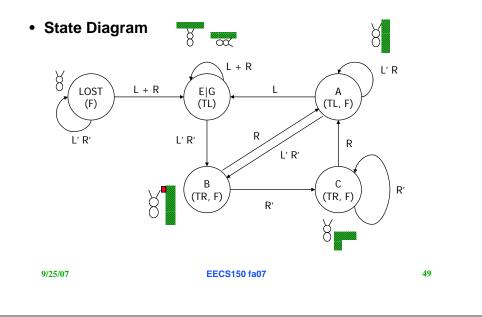
- Start-up States
  - At power-up, counter may be in an unused or invalid state
  - Designer must guarantee it (eventually) enters a valid state
- Self-starting Solution
  - Design counter so that invalid states eventually transition to a valid state
  - May limit exploitation of don't cares



**Ant Behavior** 



### **Designing an Ant Brain**



### **Synthesizing the Ant Brain Circuit**

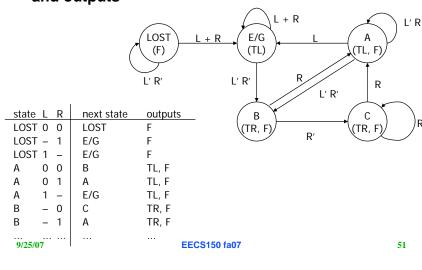
- Encode States Using a Set of State Variables
  - Arbitrary choice may affect cost, speed
- Use Transition Truth Table
  - Define next state function for each state variable
  - Define output function for each output
- Implement next state and output functions using combinational logic
  - 2-level logic (ROM/PLA/PAL)
  - Multi-level logic
  - Next state and output functions can be optimized together

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<b>Transition</b>	<b>Truth</b>	<b>Table</b>

• Using symbolic states and outputs



### **Synthesis**

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• 5 states : at least 3 state variables required (X, Y, Z) – State assignment (in this case, arbitrarily chosen)

state L R X,Y,Z	next state	outputs F TR TL + to synthesize	LOST - 000 E/G - 001 A - 010 B - 011 C - 100
000 00	000	1 0 0 these 6 functions	
000 0 1	001	100	
010 00	011	101	
010 01	010	101	
010 10	001	101	
010 11	001	101	
011 0 0	100	1 1 0	
011 0 1	010	1 1 0	
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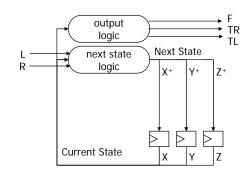
# Synthesis of Next State and Output Functions

state inputs	next state	outputs	
X,Y,Z L R	X+,Y+,Z+	F TR TL	
000 00	000	100	
000 - 1	001	100	
0001-	001	100	
00100	011	001	
001 - 1	010	001	e.g.
0011-	010	001	<b>U</b>
010 00	011	101	TR = X + Y Z
010 01	010	101	$X^{+} = X R' + Y Z R' = R' TR$
0101-	001	101	
011 - 0	100	110	
011 - 1	010	110	
100 - 0	100	110	
100 - 1	010	110	

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### **Circuit Implementation**

• Outputs are a function of the current state only -Moore machine



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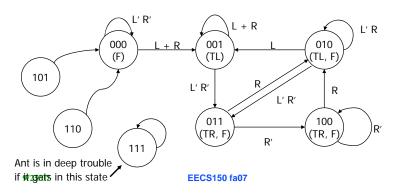
Veril	oa	Sketch
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```
module ant_brain (F, TR, TL, L, R)
  inputs
             L, R;
             F, TR, TL;
  outputs
  req
             X, Y, Z;
  assign F = function(X, Y, Z, L, R);
  assign TR = function(X, Y, Z, L, R);
  assign TL = function(X, Y, Z, L, R);
  always @(posedge clk)
    begin
      X \leq function (X, Y, Z, L, R);
      Y \leq function (X, Y, Z, L, R);
      Z \leq function (X, Y, Z, L, R);
    end
  endmodule
```

### **Don't Cares in FSM Synthesis**

- What happens to the "unused" states (101, 110, 111)?
- · Exploited as don't cares to minimize the logic
  - If states can't happen, then don't care what the functions do
  - if states do happen, we may be in trouble



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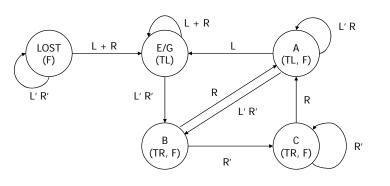
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### **State Minimization**

- Fewer states may mean fewer state variables
- High-level synthesis may generate many redundant states
- Two state are equivalent if they are impossible to distinguish from the outputs of the FSM, i. e., for any input sequence the outputs are the same
- Two conditions for two states to be equivalent:
  - 1) Output must be the same in both states
  - 2) Must transition to equivalent states for all input combinations

### **Ant Brain Revisited**

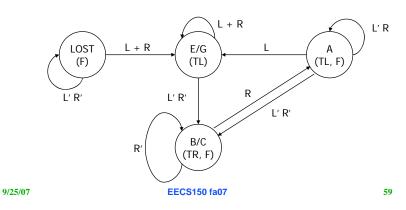
• Any equivalent states?



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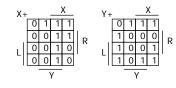
### **New Improved Brain**

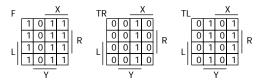
- Merge equivalent B and C states
- Behavior is exactly the same as the 5-state brain
- We now need only 2 state variables rather than 3



### **New Brain Implementation**

state	inputs	next state		outputs	
X,Y	LR	X',Y'	F	TRTL	
00	0 0	00	1	0	0
00	- 1	01	1	0	0
00	1 -	01	1	0	0
01	0 0	11	0	0	1
01	- 1	01	0	0	1
01	1 -	01	0	0	1
10	0 0	11	1	0	1
10	0 1	10	1	0	1
10	1 -	01	1	0	1
11	- 0	11	1	1	0
11	- 1	10	1	1	0





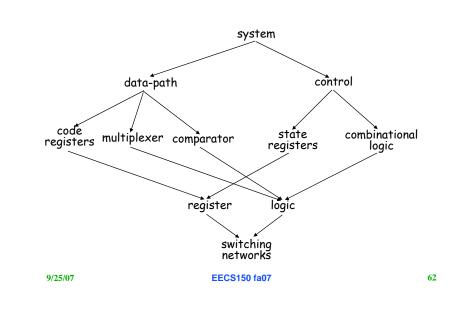
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### **Sequential Logic Implementation Summary**

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- Models for representing sequential circuits
  - Abstraction of sequential elements
  - Finite state machines and their state diagrams
  - Inputs/outputs
  - Mealy, Moore, and synchronous Mealy machines
- · Finite state machine design procedure
  - Deriving state diagram
  - Deriving state transition table
  - Determining next state and output functions
  - Implementing combinational logic

### **Design hierarchy**



### **Final Word: Blocking Vs Non-Blocking**

- · Two types of procedural assignments
  - Blocking
  - Non-Blocking
- · Why do we need them
  - Express parallelism (not straight line C)
- Synchronous system
  - All flip-flops clock data simultaneously
  - How do we express parallelism in this operation?

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Good luck on the Midterm...

### A Simple Shift Register

reg a, b, c; reg a, b, c; always @(posedge clock) always @(posedge clock) begin a = 1; a = 1; always @(posedge clock) b = a;b = a;c = bialways @(posedge clock) end c = b; **Probably not what you want!** What order does this run? reg a, b, c; reg a, b, c; always @(posedge clock) always @(posedge clock) begin a <= 1; a <= 1; always @(posedge clock) b <= a; b <= a; c <= b; always @(posedge clock) c <= b; end This works too...<sub>65</sub> This works EECS150 fa07 9/25/07

### **The Circuit**

