EECS 150 - Components and Design Techniques for Digital Systems

Lec 9 - Putting it all together... 9-25-07

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## Outline

- Top-to-bottom
- What have we covered so far?
- Combo Lock example
- FSM to logic
- Mapping to FPGAs
- Announcements
- Counters revisited
- Another example - Ant Brain


Representation of digital designs

- Physical devices (transistors, relays)
- Switches
- Truth tables
- Boolean algebra scope of CS 150
- Gates
- Waveforms
- Finite state behavior
- Register-transfer behavior
- Concurrent abstract specifications


## Logic Functions and Boolean Algebra

- Any logic function that can be expressed as a truth table can be written as an expression in Boolean algebra using the operators: ', +, and •


## Waveform View of Logic Functions

## - Just a sideways truth table

- But note how edges don't line up exactly
- It takes time for a gate to switch its output!



## An algebraic structure

- An algebraic structure consists of
- a set of elements B
- binary operations $\{+, \cdot\}$
- and a unary operation \{ ' \}
- such that the following axioms hold:

1. set $B$ contains at least two elements, $a, b$, such that $a \neq b$ 2. closure:

$$
a+b \text { is in } B
$$

$$
a \cdot b \text { is in } B
$$

3. commutativity:
4. associativity:
$a+b=b+a$
$\mathbf{a} \cdot \mathbf{b}=\mathbf{b} \cdot \mathbf{a}$
5. identity:
$a+(b+c)=(a+b)+c$ $a \cdot(b \cdot c)=(a \cdot b) \cdot c$
6. complementarity: $a+a^{\prime}=1$
$a \cdot a^{\prime}=0$

## Timing Methodologies (cont'd)

## - Definition of terms

- clock: periodic event, causes state of storage element to change; can be rising or falling edge, or high or low leve
- setup time: minimum time before the clocking event by which the input must be stable (Tsu)
- hold time: minimum time after the clocking event until which the input must remain stable (Th)
clock $\qquad$
there is a timing "window" around the clocking event during which the input must remain stable and unchanged in order to be recognized
clock $\qquad$


## Axioms \& theorems of Boolean algebra

- Identity

1. $\mathrm{X}+0=\mathrm{x}$
1D. $X \cdot 1=X$

- Null

2. $x+1=1$

- Idempotency:

3. $x+x=x$

- Involution:

4. $\left(X^{\prime}\right)$ ' $=x$

- Complementarity:

$$
\text { 5. } x+x^{\prime}=1
$$

5D. $X \cdot X^{\prime}=0$

- Commutativity:

6. $X+Y=Y+X$
6D. $X \cdot Y=Y \cdot X$

- Associativity:

7. $(X+Y)+Z=X+(Y+Z)$
7D. $(X \cdot Y) \cdot Z=X \cdot(Y \cdot Z)$
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## Axioms and theorems of Boolean algebra (cont'd)

- Distributivity:

8. $\mathrm{X} \cdot(\mathrm{Y}+\mathrm{Z})=(\mathrm{X} \cdot \mathrm{Y})+(\mathrm{X} \cdot \mathrm{Z}) 8 \mathrm{D} . \mathrm{X}+(\mathrm{Y} \cdot \mathrm{Z})=(\mathrm{X}+\mathrm{Y}) \cdot(\mathrm{X}+\mathrm{Z})$

- Uniting:

9. $X \cdot Y+X \cdot Y^{\prime}=X$
9D. $\left.(X+Y) \cdot(X+Y)^{\prime}\right)=X$

- Absorption:

10. $X+X \cdot Y=X$

10D. $X \cdot(X+Y)=X$
11D. $\left(X \cdot Y^{\prime}\right)+Y=X+Y$
12D. $X \cdot Y+X \cdot Z=$

$$
(X+Z) \cdot\left(X^{\prime}+Y\right)
$$

13D. $(X+Y) \cdot(Y+Z) \cdot(X '+Z)=$ $(\mathrm{X}+\mathrm{Y}) \cdot\left(\mathrm{X}^{\prime}+\mathrm{Z}\right)$

## Axioms and theorems of Boolean algebra (cont')

- de Morgan's:

14. $(X+Y+\ldots)^{\prime}=X^{\prime} \cdot Y^{\prime} \cdot \ldots \quad 14 D .(X \cdot Y \cdot \ldots)^{\prime}=X^{\prime}+Y^{\prime}+\ldots$

- generalized de Morgan's:

15. $\mathrm{f}^{\prime}(\mathrm{X} 1, \mathrm{X} 2, \ldots, \mathrm{Xn}, 0,1,+, \cdot)=\mathrm{f}\left(\mathrm{X} 1^{\prime}, \mathrm{X} \mathbf{2}^{\prime}, \ldots, \mathrm{Xn}\right.$ ',1,0,,+ )

- establishes relationship between • and +

Recall: What makes Digital Systems tick?


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## Sequential Logic Implementation

- Models for representing sequential circuits
- Finite-state machines (Moore and Mealy)
- Representation of memory (states)
- Changes in state (transitions)
- Design procedure
- State diagrams
- Implementation choice: counters, shift registers, FSM
- State transition table
- State encoding
- Combinational logic
» Next state functions
» Output functions


## Abstraction of State Elements

- Divide circuit into combinational logic and state
- Localize feedback loops and make it easy to break cycles
- Implementation of storage elements leads to various forms of sequential logic



## Forms of Sequential Logic

- Asynchronous sequential logic - state changes occur whenever state inputs change (elements may be simple wires or delay elements)
- Synchronous sequential logic - state changes occur in lock step across all storage elements (using a periodic waveform - the clock)



## FSM Representations

- States: determined by possible values in sequential storage elements
- Transitions: change of state
- Clock: controls when state can change by controlling storage elements

- Sequential Logic
- Sequences through a series of states
- Based on sequence of values on input signals
- Clock period defines elements of sequence


## Example: FSM Design - Combo lock

- Combination lock

Controller


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## Combo lock - controller

 implementation- Implementation of the controller


Combo Lock - State Encoding


## FSM implementation

- Steps for the hardware designer:
- Word specification
- FSM design
- Encoding
- Verification!
- At this point, hand over to synthesis tools:
- Describe FSM behavior in Verilog
- Synthesize controller
- Good encoding
- Better performance
- Fewer state bits
- Possibility of state minimization
- Tools also try to figure this out

For this example, go through the logic synthesis steps (idepally, tools take care of all thite ${ }_{5150}$ fa07

## Example: Combo Lock

| reset | new | equal | state | nstate | mux | open | Next state and output logic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (r) | ( $n$ ) | (e) | ( $s_{2} \mathrm{~s}_{1} \mathrm{~s}_{0}$ ) | $\left(n_{2} n_{1} n_{0}\right)$ | ( $m_{1} m_{0}$ ) | (0) |  |
| 1 | - | - | --- | 001 | -- | 0 |  |
| 0 | 0 | - | 001 | 001 | 01 | 0 | $\begin{aligned} & n 2=\sim r(n \text { e s1 } s 0+s 2) \\ & n 1=\sim r(n \text { e } 0+e s 1+\sim n s 1+s 2) \\ & n 0=r+s 2+n e s 1+\sim n s 0 \end{aligned}$ |
| 0 | 1 | 0 | 001 | 000 | 01 | 0 |  |
| 0 | 1 | 1 | 001 | 010 | 01 | 0 |  |
| 0 | 0 | - | 010 | 010 | 10 | 0 | mux outputs (m1, m0): |
| 0 | 1 | 0 | 010 | 000 | 10 | 0 | $\begin{aligned} & \mathrm{m} 1=s 1 \\ & \mathrm{~m} 0=s 0 \end{aligned}$open (o): |
| 0 | 1 | 1 | 010 | 011 | 10 | 0 |  |
| 0 | 0 | - | 011 | 011 | 11 | 0 |  |
| 0 | 1 | 0 | 011 | 000 | 11 | 0 | Take advantage of DCs! |
| 0 | 1 | 1 | 011 | 111 | 11 | 0 |  |
| 0 | - | - | 111 | 111 | -- | 1 |  |
| 0 | - | - | 000 | 000 | -- | 0 |  |
| 0 | - | - | 100 | --- | -- | - | How do we get these: <br> -K-maps? <br> - Tools |
| 0 | - | - | 101 | - | -- | - |  |
| 0 | - | - | 110 | --- | -- | - | -Espresso <br> > Synplicity |
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## Logic Implementation (on PLA)



Next state and output logic nextstate ( n 2 n 1 nO ):
$n 2=\sim r(n$ e s1 s0 + s2 $)$
$n 1=\sim r(n e s 0+e s 1+\sim n s 1+s 2)$ $n 0=r+s 2+n e s 1+\sim n s 0$
mux outputs ( $\mathrm{m} 1, \mathrm{mO}$ ):
$\mathrm{m} 1=s 1$
$\mathrm{mO}=\mathrm{s} 0$
open (o):

## Alternate logic implementations

- PALs
- Multi-level circuits
- Library of gates for implementation technology
- LUTs on FPGA
- ..


## Alternate Logic Representations

* Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.


How do we convert from one to the other?

## Which realization is best?

- Reduce number of inputs
- literal: input variable (complemented or not)
» approximate cost of logic gate is 2 transistors per literal
- Fewer literals means less transistors - smaller circuits
- Fewer inputs implies faster gates
- Fan-ins (\# of gate inputs) are limited in some technologies
- Reduce number of gates
- Fewer gates (and the packages they come in) means smaller circuits
- Reduce number of levels of gates
- Fewer level of gates implies reduced signal propagation delays
- How do we explore tradeoffs between increased circuit delay and size?
- Automated tools to generate different solutions
- Logic minimization: reduce number of gates and complexity
- Logic optimization: reduction while trading off against delay


## Choosing different realizations of a function



## Alternate Implementation: Controller based on Shift Register

- Previous implementation
- Comb. logic as gates (PLA)
- State bits in latches


Controller using Shift Register

| reset | new | equal | state | nstate | mux | open |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (r) | (n) | (e) | ( $s_{3} s_{2} s_{1} s_{0}$ ) | $\left(n_{3} n_{2} n_{1} n_{0}\right)$ | $\left(m_{1} m_{0}\right)$ | (0) |  |
| 1 | - | - | ---- | 1000 | -- | 0 |  |
| 0 | 0 | - | 1000 | 1000 | 01 | 0 | One-hot encoding scheme: state |
| 0 | 1 | 0 | 1000 | 0000 | 01 | 0 | transition is a shift right |
| 0 | 1 | 1 | 1000 | 0100 | 01 | 0 | Mux control: |
| 0 | 0 | - | 0100 | 0100 | 10 | 0 | $C 1=01, C 2=10, C 3=11$ (pre- |
| 0 | 1 | 0 | 0100 | 0000 | 10 | 0 | established) |
| 0 | 1 | 1 | 0100 | 0010 | 10 | 0 |  |
| 0 | 0 | - | 0010 | 0010 | 11 | 0 | State encoding: |
| 0 | 1 | 0 | 0010 | 0000 | 11 | 0 | S1 $=1000, \mathrm{~S} 2=0100$, |
| 0 | 1 | 1 | 0010 | 0001 | 11 | 0 | S3 $=0010$, OPEN $=0001$, Error $=$ 0000 |
| 0 | - | - | 0001 | 0001 | -- | 1 |  |
| 0 | - | - | 0000 | 0000 | -- | 0 | Output encoding: |
| 0 | - | - | 11-- | ---- | -- | - | Closed $=0$, Open $=1$ |
| 0 | - | - | 1-1- | ---- | -- | - | ERR |
| 0 | - | - | 1--1 | ---- | -- | - | enam |
| 0 | - | - | -11- | ---- | -- | - |  |
| 0 | - | - | -1-1 | ---- | -- | - | cosed |
| 0 | $9 / 25 / 07$ | - | --11 | -- | CS150 fa07 | - | $29$ |

Combo lock controller on shift reg


## How does the combo lock look on an

 FPGA?- Latches
- implement shift register (chain of 4 latches)
- LUTs
- Combinational logic for out and mux control
- Routing fabric
- Connect logical nets between CLBs


## Inside the FPGA

- Network of Combinational logic blocks, memory and I/O
- rich interconnect network
- special units - multipliers, carry-logic
- CLBs
- 3 or 4-input look up table (LUT)
- implements combinational logic functions
- Register optionally stores output of LUT
- Logic on FPGA
- Configure LUTs (table of entries)
- Configure latches in CLB
- Program interconnect



## LUT as general logic gate

## User Programmability



- Latch-based (Xilinx, Altera, ...)

+ reconfigurable
- volatile
- relatively large.
- Latches are used to:

1. make or break cross-point connections in the interconnect
2. define the function of the logic blocks
3. set user options:
» within the logic blocks
» in the input/output blocks
» global reset/clock

- "Configuration bit stream" can be loaded under user control:

All latches are strung together in a shift chain:

## 4-LUT Implementation



Configuring CLBs


out $=\sim(A 1$ A2 A3)

## Configuring Routes



Announcements/Reminders

- First mid term - Thursday 9/27
- No notes (... to discuss)
- Review materials are in the HW4
- Review session tonight 8-10 642-WALK (9255)
- Trying to make the exams routine
- Feel free to approach us with questions...
- No discussion Thurs, yes friday
- Lab 5 - Where's the music?
- Normal lab lecture on Friday


## Sequential Systems - more examples

- Beat the combo lock example to death
- Direct FSM implementation
- Shift register
» Multiple logic representations
» gates to LUTs
- Up next
- A few quick counter examples
- Another design problem - Ant Brain


## Can Any Sequential System be

 Represented with a State Diagram?- Shift Register
- Input value shown on transition arcs
- Output values shown within state node



## Counter Example

- Shift Register
- Input determines next state

| In | C 1 | C 2 | C 3 | N 1 | N 2 | N 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
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N1: = In
N2: $=\mathrm{Cl}$
N3: C 2

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## Counters are Simple Finite State Machines

- Counters
- Proceed thru well-defined state sequence in response to enable
- Many types of counters: binary, BCD, Gray-code
- 3-bit up-counter: 000, 001, 010, 011, 100, 101, 110, 111, 000, ...
- 3-bit down-counter: 111, 110, 101, 100, 011, 010, 001, 000, 111, ...

module binary_upcntr (q, clk)
inputs clk;
$\begin{array}{ll}\text { outputs } & {[2: 0]} \\ \text { reg ; } \\ {[2: 0]}\end{array}$
reg [2:0] q, p;
always @(q) // Next state
case (q)
3'b000: p = 3'b001.
3'b001: p = 3'b010;
3'b111: $p=3^{\prime} b 000 ;$
endcase
always @(posedge clk) // Update state q <= p;


## More Complex Counter Example

- Complex Counter
- Repeats five states in sequence
- Not a binary number representation
- Step 1: Derive the state transition diagram
- Count sequence: 000, 010, 011, 101, 110
- Step 2: Derive the state transition table from the state transition diagram


| Present State |  |  |  | Next State |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | B | A | C+ | B+ | A+ |  |
| 0 | 0 | 0 | 0 | 1 | 0 |  |
| 0 | 0 | 1 | - | - | - |  |
| 0 | 1 | 0 | 0 | 1 | 1 |  |
| 0 | 1 | 1 | 1 | 0 | 1 |  |
| 1 | 0 | 0 | - | - | - |  |
| 1 | 0 | 1 | 1 | 1 | 0 |  |
| 1 | 1 | 0 | 0 | 0 | 0 |  |
| 1 | 1 | 1 | - | - | - |  |

9/25/07 note the don't care conditions that arise from the unused state codes

## More Complex Counter Example (cont'd)

- Step 3: K-maps for Next State Functions


$$
\mathrm{C}+:=\mathrm{A}
$$

$$
\mathrm{B}+:=\mathrm{B}^{\prime}+\mathrm{A}^{\prime} \mathrm{C}^{\prime}
$$

$$
\mathrm{A}+:=\mathrm{BC}
$$

## Self-Starting Counters (cont'd)

- Re-deriving state transition table from don't care assignment



## Self-Starting Counters

- Start-up States
- At power-up, counter may be in an unused or invalid state
- Designer must guarantee it (eventually) enters a valid state
- Self-starting Solution
- Design counter so that invalid states eventually transition to a valid state
- May limit exploitation of don't cares



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Ant Behavior


## Designing an Ant Brain

## - State Diagram 8 oo



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## Transition Truth Table

- Using symbolic states and outputs


| state $L$ | $R$ | next state | outputs |  |
| :--- | :--- | :--- | :--- | :--- |
| LOST | 0 | 0 | LOST | F |
| LOST | - | 1 | E/G | F |
| LOST | 1 | - | E/G | F |
| A | 0 | 0 | B | TL, F |
| A | 0 | 1 | A | TL, F |
| A | 1 | - | E/G | TL, F |
| B | - | 0 | C | TR, F |
| B | - | 1 | A | TR, F |
| $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ |

## Synthesizing the Ant Brain Circuit

- Encode States Using a Set of State Variables
- Arbitrary choice - may affect cost, speed
- Use Transition Truth Table
- Define next state function for each state variable
- Define output function for each output
- Implement next state and output functions using combinational logic
- 2-level logic (ROM/PLA/PAL)
- Multi-level logic
- Next state and output functions can be optimized together


## Synthesis

- 5 states : at least 3 state variables required (X, Y, Z)
- State assignment (in this case, arbitrarily chosen)



## Synthesis of Next State and Output Functions

| state inputs $X, Y, Z \quad L \quad R$ | next state $\mathrm{X}^{+}, \mathrm{Y}^{+}, \mathrm{Z}^{+}$ | outputs <br> F TR TL |  |
| :---: | :---: | :---: | :---: |
| OOO 00 | 000 | 100 |  |
| 000-1 | 001 | 100 |  |
| 0001 - | 001 | 100 |  |
| 00100 | 011 | 001 |  |
| 001-1 | 010 | 001 | e.g. |
| 001 1- | 010 | 001 |  |
| 01000 | 011 | 101 | TR $=X+Y \mathrm{Z}$ |
| 01001 | 010 | 101 | $\mathrm{X}^{+}=\mathrm{X} \mathrm{R}^{\prime}+\mathrm{Y} Z \mathrm{R}^{\prime}=\mathrm{R}^{\prime} \mathrm{TR}$ |
| 0101 - | 001 | 101 |  |
| 011-0 | 100 | 110 |  |
| 011-1 | 010 | 110 |  |
| 100-0 | 100 | 110 |  |
| 100-1 | 010 | 110 |  |

## Circuit Implementation

- Outputs are a function of the current state only Moore machine



## Verilog Sketch

```
module ant_brain (F, TR, TL, L, R)
    inputs
    outputs F, TR, TL;
    reg X, Y, Z;
    assign F = function(X, Y, Z, L, R);
    assign TR = function(X, Y, Z, L, R);
    assign TL = function(X, Y, Z, L, R)
    always @(posedge clk)
        begin
            X <= function (X, Y, Z, L, R);
            Y <= function (X, Y, Z, L, R);
            Z <= function (X, Y, Z, L, R);
        end
    endmodule
```


## State Minimization

- Fewer states may mean fewer state variables
- High-level synthesis may generate many redundant states
- Two state are equivalent if they are impossible to distinguish from the outputs of the FSM, i. e., for any input sequence the outputs are the same
- Two conditions for two states to be equivalent:
- 1) Output must be the same in both states
- 2) Must transition to equivalent states for all input combinations

New Brain Implementation

## Ant Brain Revisited

- Any equivalent states?


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- Merge equivalent B and C states
- Behavior is exactly the same as the 5 -state brain
- We now need only 2 state variables rather than 3


## New Improved Brain



| state |  | next state outputs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| X,Y | L | X', $\mathrm{Y}^{\prime}$ |  | TR | RTL |
| 00 | 0 | 00 |  | 0 | 0 |
| 00 | - | 01 |  | 0 | 0 |
| 00 | 1 | 01 |  | 0 | 0 |
| 01 | 0 | 11 |  | 0 | 1 |
| 01 | - | 01 |  | 0 | 1 |
| 01 | 1 | 01 |  | 0 | 1 |
| 10 | 0 | 11 |  | 0 | 1 |
| 10 | 0 | 10 |  | 0 | 1 |
| 10 | 1 | 01 |  | 0 | 1 |
| 11 |  | 11 |  | 1 |  |
| 11 | - | 10 |  | 1 |  |



## Sequential Logic Implementation Summary

## Design hierarchy

- Models for representing sequential circuits
- Abstraction of sequential elements
- Finite state machines and their state diagrams
- Inputs/outputs
- Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
- Deriving state diagram
- Deriving state transition table
- Determining next state and output functions
- Implementing combinational logic


Final Word: Blocking Vs Non-Blocking

- Two types of procedural assignments
- Blocking
- Non-Blocking
- Why do we need them
- Express parallelism (not straight line C)
- Synchronous system
- All flip-flops clock data simultaneously
- How do we express parallelism in this operation?


## A Simple Shift Register

reg a, b, c;
always @(posedge clock) begin

$$
\begin{aligned}
& a=1 \\
& b=a \\
& c=b
\end{aligned}
$$

end
Probably not what you want!
reg $a, b, c$;
always @(posedge clock)
begin
$\mathrm{a}<=1 ;$
$\mathrm{b}<=\mathrm{a} ;$
$\mathrm{c}<=\mathrm{b} ;$
end

This works
reg a, b, c;
always @(posedge clock) a = 1;
always @(posedge clock)

$$
b=a ;
$$

always @(posedge clock) $\mathrm{c}=\mathrm{b}$;
What order does this run?
reg a, b, c;
always @(posedge clock) a <= 1;
always @(posedge clock) b <= a;
always @(posedge clock)
c <= b;
This works too... 65

## The Circuit



Non-Blocking: RHS computed at beginning of execution instance. 9/25/07

