## Outline

O Last time:
$\rightarrow$ Asynchronous Circuits
$\rightarrow$ Moore and Mealy Standard Forms
$\rightarrow$ Design Example: Word Problem
O This lecture:
$\rightarrow$ Race-free State Assignment
$\rightarrow$ Excitation Equations
$\rightarrow$ Design Example: Implementation
$\rightarrow$ Summary of Asynchronous Design Process

## Steps to Asynchronous FSM Design

Construct a Primitive Flow Table from the word statement of the problem.
$\checkmark$ Derive a minimum-row primitive flow table or Reduced Primitive Flow Table by eliminating redundant, stable total-states.
O Convert the resulting table to Mealy form, if necessary, so that the output value is associated with the total state rather than the internal state.
O Derive a minimum-row flow table, or Merged Flow Table, by merging compatible rows of the reduced primitive flow table using a merger diagram. (Note: solution not necessarily unique)
O Perform race-free, or critical-race-free, state assignment, adding additional states if necessary.
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O Draw logic diagram that shows ideal combinational next-state and output functions as well as necessary delay elements.

## Conversion to Mealy Form for Merging: Example 1

|  | $\mathbf{X}_{1} \mathbf{X}_{2}$ |  | Z |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0001 | 1110 | 20 | 01 | 11 | 10 |
| 1 | (1) 2 | 3 | 0 | - | - | - |
| 2 | 1 (2) | 4 - | - | 0 | - | - |
| 3 | 1 - | 53 | - | - | - | 0 |
| 4 | - 6 | 4. 3 | - | - | 1 | - |
| 5 | - 6 | (5) 3 | - | - | 0 | - |
| 6 | 16 | 5 |  |  |  | - |

## Create Merger Diagram



What we are doing is choosing internal states that can have the same code, though the total states will still be different (since they will not have the same code for the same inputs; if that were possible they would have been redundant).
Two rows of a reduced primitive Mealy flow table are compatible and can be merged into a single row iff there are no state or output conflicts in any column.
Draw a line between any pair of rows (states) which are compatible and can be merged.

Choose the sub-groups of fully-connected rows that will result in the maximum reduction. In this case, $\{1,2\}$ and $\{3,5,6\}$ are best. We could also have chosen $\{1,3\}$ and $\{5,6\}$, or just $\{3,6\}$ in which case no others would be permitted.

## Example 1: After Merging

|  | x1x2 |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| $\mathrm{a}(1,2)$ | 1 | 2 | 4 | 3 | 0 | 0 | - | - |
| $\mathrm{b}(3,5,6)$ | 1 | 6 | 5 | 3 | - | 0 | 0 | 0 |
| $\mathrm{c}(4)$ | - | 6 | 4 | 3 | - | - | 1 | - |

## Conversion to Mealy Form for Merging:

 Example 2| CS |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Z | 00 | 01 | 11 | 10 | 00 | 01 | 11 |
| 10 |  |  |  |  |  |  |  |  |
| 1 | 1 | 3 | - | 2 | 0 | - | - | - |
| 2 | 1 | - | 6 | 2 | - | - | - | 0 |
| 3 | 1 | 3 | 4 | - | - | 0 | - | - |
| 4 | - | 3 | 4 | 5 | - | - | 1 | - |
| 5 | 1 | - | - | 5 | - | - | - | 1 |
| 6 | - | 3 | 6 | - | - | - | 0 | - |

## Create Merger Diagram: Example 2



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## Example: After State Reduction and Merging

|  | 80 | 01 | 11 | 10 | 00 | 01 |  |  | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}(1,2)$ | (1) | (2) | 4 | 3 | 0 | 0 |  |  |  |
| $b(3,5,6)$ | 1 | (6) | (5) | (3) | - | 0 | 0 |  | 0 |
| c (4) | - | 6 | (4) | 3 | - | - | 1 |  | - |

## Example 1: States Labelled in Terms of Internal States

|  | X1X2 |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
|  | a | a | c | b | 0 | 0 | - | - |
| a | a | b | b | b | - | 0 | 0 | 0 |
| b | - | b | c) | b | - | - | 1 | - |

## Example: Race-Free State Assignment



O Line between states indicates required transition.
O Need assignment such that only one state variable changes during each state transition


## Example: Critical-Race-Free State Assignment

|  | X1X2 |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |  |
|  |  |  |  |  |  |  |  |  |  |
| a 00 | 00 | 00 | 10 | 01 | 0 | 0 | - | 0 v |  |
| b 01 | 00 | 01 | 01 | 01 | 0 u | 0 | 0 | 0 |  |
| d 11 | - | 01 | - | - | - | -s | - | - |  |
| c 10 | - | 11 | 10 | 00 | - | 1 t | 1 | - |  |

O In col. 01, row C, 10->11->01: critical race avoided
O In col 10, could have directed transition via row d but already have transition to b in row a , so use it.
O Fill in outputs corresponding to unstable states to avoid momentary false outputs during transitions

## Critical-Race-Free State Assignment via Shared-Row Assignments

|  | 00 | 01 | 11 | 10 |
| :--- | :---: | :---: | :---: | :---: |
| a | 1 | 2 | 5 | 4 |
| b | 7 | 2 | 3 | 10 |
| c | 1 | 8 | 3 | 4 |
| d | 7 | 8 | 5 | 4 |
| e | 1 | 9 | 5 | 6 |
| f | $(11)$ | 8 | 3 | 6 |

$\bigcirc$ Required transitions:
col 00: e,c -> a; d->b
col 01: a->b; c,f->d
col 11: b,f->c; a,d->e
col 10: a,c->d; e->f

## Shared Row Assignment

O Consider required transitions: e,c -> a
O Could implement as: e->a, c->a; or e->c->a; or c->e->a; or any of the above going through an intermediate state (or states).
O To avoid critical races, must ensure that $\{\mathrm{a}, \mathrm{c}, \mathrm{e}\}$ are placed in adjacent squares on the assignment map. Similar for other constraints. Must satisfy:

$$
\{\mathrm{a}, \mathrm{c}, \mathrm{e}\},\{\mathrm{b}, \mathrm{~d}\},\{\mathrm{a}, \mathrm{~b}\},\{\mathrm{c}, \mathrm{~d}, \mathrm{f}\},\{\mathrm{b}, \mathrm{c}, \mathrm{f}\},\{\mathrm{a}, \mathrm{~d}, \mathrm{e}\},\{\mathrm{a}, \mathrm{c}, \mathrm{~d}\},
$$ $\{\mathrm{e}, \mathrm{f}\}$

| a | e |
| :--- | :--- |
| b | d |
| x | c |
| y | f |

## Completion of the Output Table

 x1X2|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| a 00 | 00 | 00 | 10 | 01 | 0 | 0 | - | 0 v |
| b 01 | 00 | $(01)$ | $01)$ | 01 | 0 u | 0 | 0 | 0 |
| d 11 | - | 01 | - | - | - | -s | - | - |
| c 10 | - | 11 | $(10$ | 00 | - | 1 t | 1 | - |

From State 00 , input change $00->10$ causes transition to stable state 01 . To avoid glitch if logic synthesis assigns "don't care" in output to 1 for input value 10 , make it a 0 (labelled v above).

From State 01 , input change $01->00$ causes transition to State 00 , so avoid possible glitch by assigning output in State 01 for input 00 to 0 (u above)
From State 10 , input change $11->01$ causes transition to stable state 01 , via 11. Since output goes from 1 to 0 , choose output at $t$ above to be 1 , consistent with the starting value, but leave output at s a "don't care" since must make the transition somewhere and either before or after State 11 is the same.

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