Outline

- Last time:
  - Quiz Review
- This lecture:
  - Adders and subtracters: Half adder, full adder
  - Adder organizations: ripple, carry look-ahead (CLA), carry bypass
  - Full adder as a full subtracter
  - Sign-magnitude
  - One's & Two's Complement
  - BCD, Floating-point

Bits, Bytes, Nibbles and Words: Some Definitions

- Data types which are ordered lists of binary digits often have the following names:
  - Bit: 1 digit
  - Nibble: 4 digits
  - Byte: 8 digits
  - Word: The standard memory bus width in your architecture (e.g. 16-bit, 32-bit, 64-bit words).

Addition

- **Decimal Addition:**

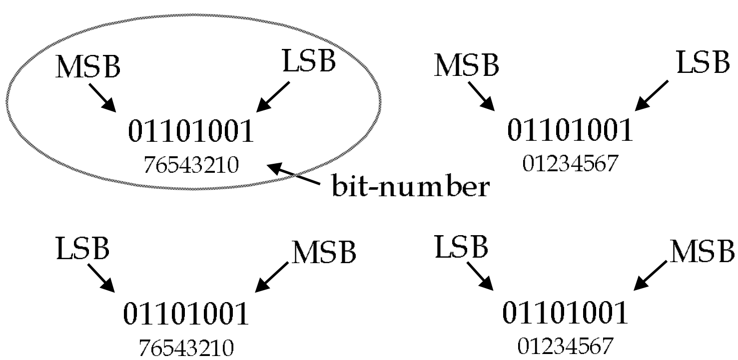
```
  1990
+ 215
------
```

- **Binary Addition:**

```
 10101   ------>    21
+ 110    ------>   + 6
------             ----
         <------
```

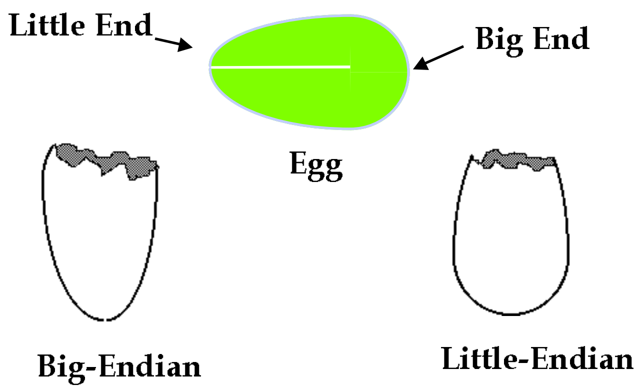

Bit-Ordering, Byte-Ordering and the Endian Wars

- The ordering and naming of digits in a number (binary or decimal) is an arbitrary choice. However, it would be useful if we all agreed on the same choice!

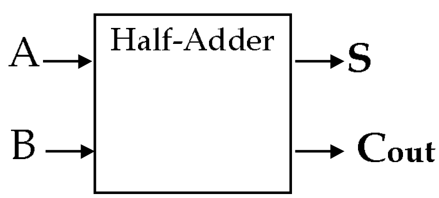


Bit-Ordering, Byte-Ordering and the Endian Wars

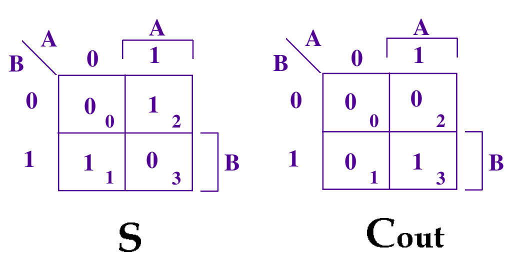
- In his travels, Gulliver came across the warring races of Big-Endians and Little-Endians. Dr. Danny Cohen (ISI) uses this situation as an analogy:



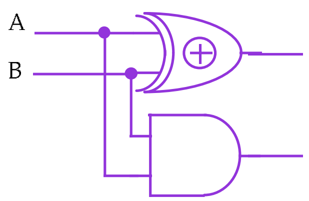
The Addition of Two Binary Digits: The Half-Adder



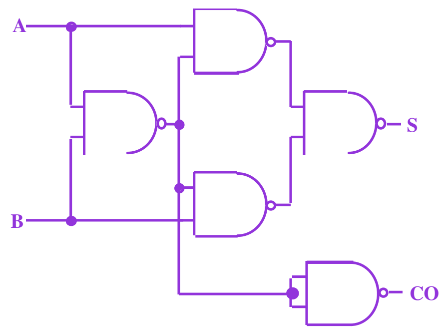
AB	S	Cout
0 0	0	0
0 1	1	0
1 0	1	0
1 1	0	1



Half Adder Implementation



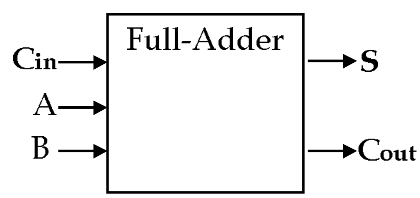
(a) Any gates

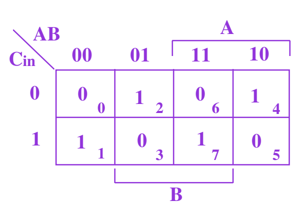


(b) Two-input NANDS

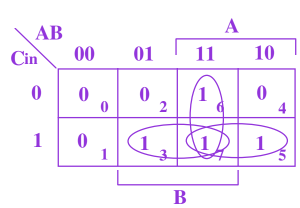
Addition of Two Binary Digits and a Carry-In: The Full Adder

Cin	A	B	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





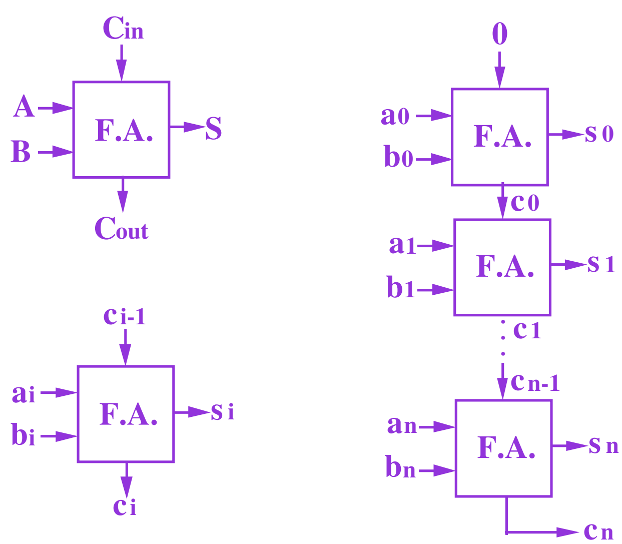
S



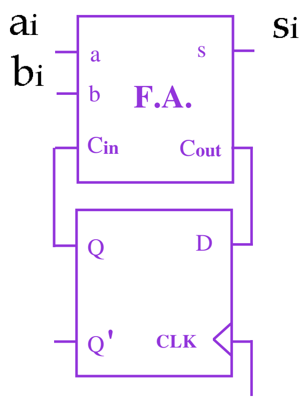
Cout

Full-Adder Implementation

The Ripple-Carry n-Bit Binary Parallel Adder



Serial Adder



Subtraction

- **Decimal Subtraction:**

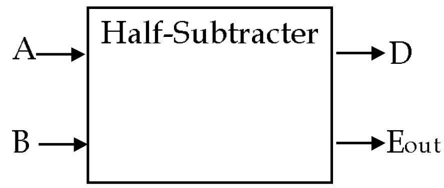
Minuend: 1990
Subtrahend: - 215

- **Binary Subtraction:**

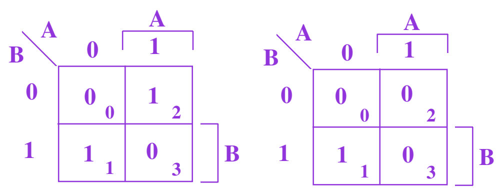
10101 → 21
- 110 → - 6

The Subtraction of Two Binary Digits: The Half-Subtracter

Perform:
A-B



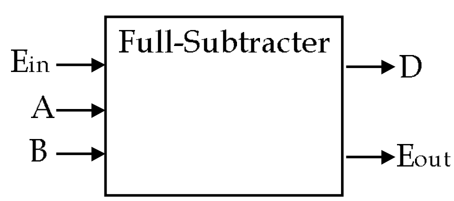
AB	D	Eout
0 0	0	0
0 1	1	1
1 0	1	0
1 1	0	0

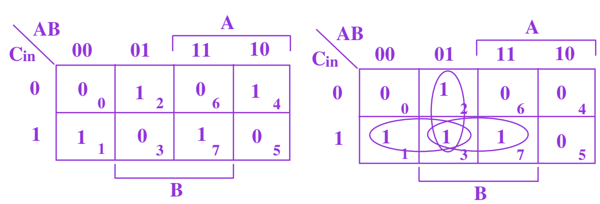


D Eout

Subtraction of Two Binary Digits and a Borrow-In: The Full Subtracter

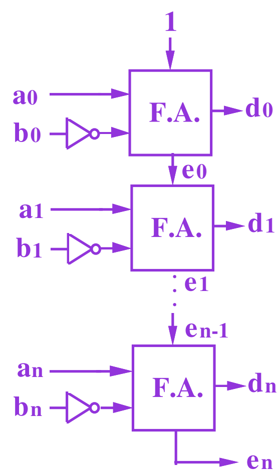
Ein	A	B	D	Eout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1





D Eout

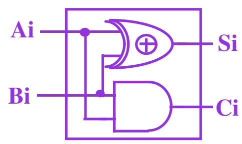
Ripple Subtracter



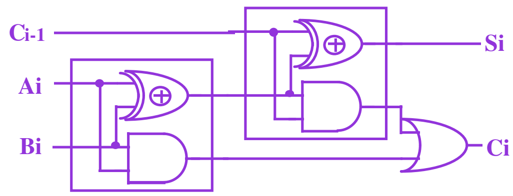
Subtraction & Number Systems

- But what about if the result of the subtraction is itself negative? How would we know? How do we represent negative numbers?
  - Signed-magnitude number system
  - Signed-1's-complement number system
  - Signed-2's-complement number system

Full-Adder from Half-Adders



Half-adder



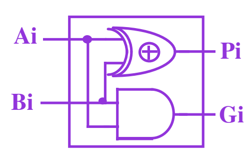
Carry Look-Ahead Addition

A	B	Cin	S	Cout	
0	0	0	0	0	No carry generated
0	0	1	1	0	No carry generated
0	1	0	1	0	Carry **P**ropagated
0	1	1	0	1	Carry **P**ropagated
1	0	0	1	0	Carry **P**ropagated
1	0	1	0	1	Carry **P**ropagated
1	1	0	0	1	Carry **G**enerated
1	1	1	1	1	Carry **G**enerated

$G_i = A_i B_i$

$P_i = A_i \oplus B_i$

$\Rightarrow C_i = G_i + P_i C_{i-1}$



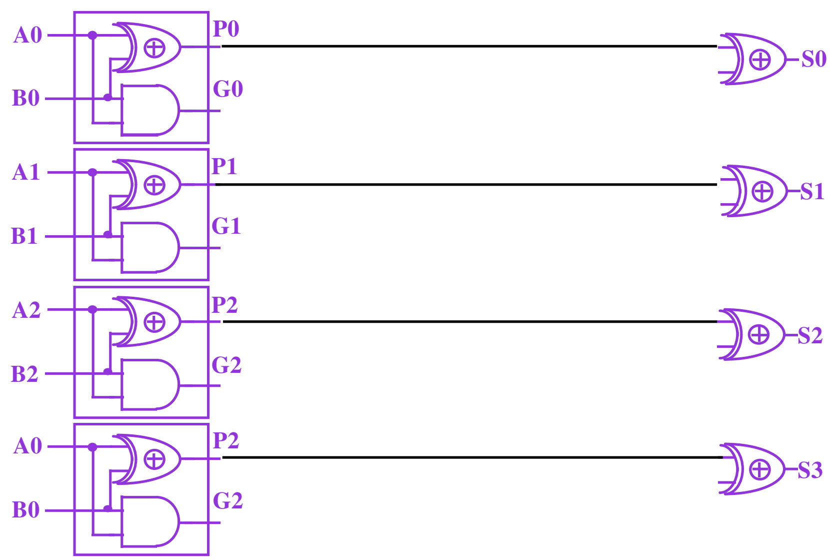
4-Bit Carry Look-Ahead Adder

$G_i = A_i B_i$
$P_i = A_i - B_i$

$\Rightarrow C_i = G_i + P_i C_{i-1}$

$C_0 = G_0 + P_0 C_{-1} = G_0$
$C_1 = G_1 + P_1 C_0 = G_1 + P_1 G_0$
$C_2 = G_2 + P_2 C_1 = G_2 + P_2 (G_1 + P_1 C_0) = G_2 + P_2 G_1 + P_2 P_1 C_0$
$C_3 = G_3 + P_3 C_2 = G_3 + P_3 (G_2 + P_2 (G_1 + P_1 C_0))$
$\quad = G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 C_0)$
$\quad = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 C_0)$

4-Bit Carry Look-Ahead Adder



Sign-Magnitude Representation: 4-bits

- Comments:
  - For n-bit word, left-most bit is sign-bit, 0 for+ve number & 1 for -ve number, and remaining n-1 bits represent integer magnitude.
  - Represents both + and - zero
  - Logic networks to process sign-magnitude awkward to build, so not used often.

0110 =
1001 =
0000 =
1000 =
1111 =
0111 =

0110

sign
bit: +0, -1

binary magnitude

Two's Complement Representation (N*): 4 bits

For n-bit positive integer N, define N* = 2^n -N

N	N*
0110	=
0001	=
0000	=
0111	=

N *	D
1111	=
1000	=
0000	=
0111	=

- Comments:
  - For n-bit word, positive number in 2's complement form is represented by a zero followed by the binary magnitude as in sign-magnitude.
  - Single representation of zero

One's Complement Representation (N'): 4 bits

For n-bit positive integer N, define N' =(2^n - 1) -N

N		N'
0110	=	
0001	=	
0000	=	
0111	=	

N '		D
1111	=	
1000	=	
1101	=	
0000	=	
0111	=	

- Comments:
  - For n-bit word, positive number in 1's complement form is represented by a zero followed by the binary magnitude as in sign-magnitude and 2's complement.
  - Representation of +ve and -ve zero
  - 1's complement can also be formed by compelementing all the bits in N

1's and 2's Complement

- N* = 2^n-N = (2^n - 1 - N) +1 = N' +1
- N = 2^n-N* and N = (2^n-1) - N'
  - Easy way to get N*: start at right-hand-end and complement all bits to the left of the first 1.
  - Concept can be extended to the "R's complement" of a number in Base R.
    - Consider R=8 with n=4; the 8's complement of 03458 = 8^4 - 0345 = 74328

Signed Binary Integers (word-length =4): Sign-magnitude, 1's complement, 2's complement

decimal	all systems
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	N/A

decimal	sign & mag	2's comp. N*	1's comp. N'
-0	1000	N/A	1111
-1	1001	1111	1110
-2	1010	1110	1101
-3	1011	1101	1100
-4	1100	1100	1011
-5	1101	1011	1010
-6	1110	1010	1001
-7	1111	1001	1000
-8	N/A	1000	N/A

Addition of 2's Complement Numbers

(a) Positive numbers, sum < 2^(n-1)

```
 +3  0011
 +4  0100
 ---------
 +7  0111 (correct answer)
```

(b) Positive numbers, sum >= 2^(n-1)

```
 +5  0101
 +6  0110
 ---------
     1001 (wrong since overflow)
```

(c) Pos + Neg numbers, Neg > Pos

```
 +5  0101
 -6  1010
 ---------
 -1  1111
```

(d) Pos + Neg numbers, Pos > Neg

```
 -5  1011
 +6  0110
 ---------
 +1  0001 (correct if carry ignored)
```

(e) Negative no's. |sum| <= 2^(n-1)

```
 -3  1101
 -4  1100
 ---------
 -7  1001 (correct if carry ignored)
```

(f) Negative no's. |sum| > 2^(n-1)

```
 -5  1011
 -6  1010
 ---------
     0101 (wrong since overflow)
```

Addition of 1's Complement Numbers

- Same as 2's complement except that instead of discarding the last carry, it is added to the n-bit sum in the right-most position ("end-around" carry).

(a) & (b) same as 2's complement

(c) Pos + Neg numbers, Neg > Pos

```
+5    0101
-6    1001
-----------
-1    1110 (correct answer)
```

(d) Pos + Neg numbers, Pos > Neg

```
-5    1010
+6    0110
 (1)  0000
         1 (end-around carry)
-----------
+1    0001 (correct; no overflow)
```

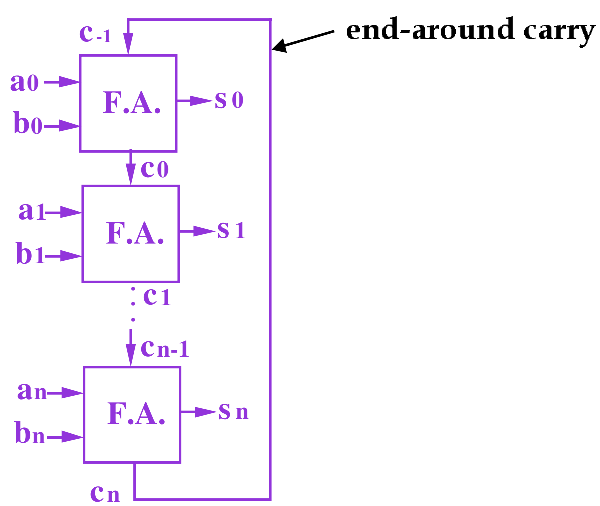
(e) Negative no's. |sum| <= 2^(n-1)

```
-3    1100
-4    1011
 (1)  0111
         1 (end-around carry)
-----------
-7    1000 (correct; no overflow)
```

(f) Negative no's. |sum| > 2^(n-1)

```
-5    1010
-6    1001
 (1)  0011
         1 (end-around carry)
-----------
      0100 (wrong answer; overflow)
```


One's Complement Adder



Redundant Representations of Numbers

- Normal way of thinking of arithmetic circuit design: "Given a set of binary numbers in 1's complement, 2's complement, etc., find the fastest/smallest circuit that implements the arithmetic functions."
- Another way of asking the questions: "Given that we want the faster/smallest arithmetic circuits, how should the binary numbers be represented?"
  - ➜ Should we use log2(N) bits to represent N numbers?