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EECS150
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Quiz \#3 - Solution

There are many possible solutions - we present 3 plausible ones. There are two preliminary points which are important in all solutions:

- $\mathbf{s = 1}$. Since this is a combinational circuit, not a sequential one, we do not need the flip flop. In all cases, the output multiplexor should pass on the signal which bypasses the flip flop (i.e. $\mathrm{s}=1$ ).
- Decompose the 8 -input OR. A single CLB cannot possibly implement the 8 -input OR which generates $y$. You must decompose that OR gate. This is easy, since OR is associative: $a+b+c=(a+b)+c=a+(b+c)$. That means a wide OR can be implemented by a sequence of smaller ORs as well as by a tree (as in Homework \#2, problem 2, which did the same for a wide XOR). To minimize total CLB usage, the particular OR decomposition you choose should depend on the partitioning of the rest of the circuit.

Recall that this is our CLB:


The easiest way to use the CLB is as a 4-LUT. We compose the 4-LUT from the pair of 3-LUTs by feeding the same 3 inputs into $\{b, c, d\}$ as into $\{e, f, g\}$ and by using $a$ for the fourth input. The first 3-LUT implements the 4-LUT function assuming $a=0$ while the second assumes $a=1$. This technique is demonstrated in Homework \#3, problem 3.

A very simple partitioning can be done using 114 -LUTs. Each of the 8 AND-AND terms uses one CLB, and the OR is partitioned into a tree of 3 CLBs.


| a | b | c | d | e | f | g | s | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x3 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t9 |
| x4 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t10 |
| x5 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t11 |
| x6 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t12 |
| x7 | x0 | x1 | x 2 | x0 | x1 | x2 | 1 | t13 |
| x8 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t14 |
| x9 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t15 |
| x10 | x0 | x1 | x2 | x0 | x1 | x2 | 1 | t16 |
| t12 | t9 | t10 | t11 | t9 | t10 | t11 | 1 | w0 |
| t16 | t13 | t14 | t15 | t13 | t14 | t15 | 1 | w1 |
| 0 | w0 | w1 | - | - | - | - | 1 | y |

Implementation in 11 CLBs (4-LUT configuration).

The following partition in 5 CLBs is due to Drew Pertula. The 8 AND-AND terms can be grouped into 4 pairs, where the difference between each pair is the inversion of the x 2 input. This guarantees that, in any such pair, one AND-AND term is forced to zero. Since the AND-AND terms are subsequently OR-ed, the value of x2 effectively selects which among each pair of AND-AND terms will pass to the output. Now we can pack each pair of OR-ed AND-AND terms into a single CLB - each AND-AND (now without x 2 as an input) gets a 3-LUT, and x 2 controls the multiplexer to select one of them. This requires 4 CLBs ( 2 AND-ANDs in each), and one additional CLB to OR their outputs (using a 4-LUT configuration). Total: 5 CLBs.


| a | b | c | d | e | f | g | s | h |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| x 2 | x 0 | x 1 | x 3 | x 0 | x 1 | x 4 | 1 | w 0 |
| x 2 | x 0 | x 1 | x 5 | x 0 | x 1 | x 6 | 1 | w 1 |
| x 2 | x 0 | x 1 | x 7 | x 0 | x 1 | x 8 | 1 | w 2 |
| x 2 | x 0 | x 1 | x 9 | x 0 | x 1 | x 10 | 1 | w 3 |
| w 3 | w 0 | w 1 | w 2 | w 0 | w 1 | w 2 | 1 | y |

Implementation in 5 CLBs.

We can do one better - 3 CLBs. You must first realize that the circuit is an 8-to-1 multiplexor. It chooses one of $\{\mathrm{x} 3, \ldots, \mathrm{x} 10\}$ according to the select bus $\{\mathrm{x} 2, \mathrm{x} 1, \mathrm{x} 0\}$. The first column of ANDs implements a decoder, converting the binary number $\{\mathrm{x} 2, \mathrm{x} 1, \mathrm{x} 0\}$ into a one-hot representation $\{\mathrm{t} 1, \ldots, \mathrm{t} 8\}$ - only one of $\{\mathrm{t} 1, \ldots, \mathrm{t} 8\}$ will be 1 , the rest 0 . The second column masks out the unselected inputs from $\{\mathrm{x} 3, \ldots, \mathrm{x} 10\}$ by AND-ing them with 0 - only one of $\{t 9, \ldots, t 16\}$ will actually copy an $x$ input, the rest will be 0 . The final OR simply passes-on whichever result was selected.

An 8-to-1 multiplexor can be implemented by a tree of 2-to-1 multiplexors. A 2-to-1 multiplexor fits in a 3-LUT. A CLB combines 2 2-to-1 muxes into a 4 -to- 1 mux. A pair of such CLBs fed through a 2-to-1 multiplexor in a third CLB forms an 8-to-1 mux.


| a | b | c | d | e | f | g | S | h |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| x 1 | x 3 | x 4 | x 2 | x 5 | x 6 | x 2 | 1 | t 4 |
| x 1 | x 7 | x 8 | x 2 | x 9 | x 10 | x 2 | 1 | t 5 |
| 0 | t 4 | t 5 | x 0 | - | - | - | 1 | y |

Implementation in 3 CLBs.

