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Homework #7 – Solution

The following solutions are reproduced from the Katz textbook's solutions manual.

2.7

a.) $(X + Y)(X + \overline{Y}) = X$
$= (X + Y)(X + \overline{Y})$
$= X X + X \overline{Y} + X Y + Y \overline{Y}$
$= X + X(\overline{Y} + Y) + 0$
$= X + X(1)$
$= X$

b.) $X(X + Y) = X$
$= X(X + Y)$
$= XX + X Y$
$= X + XY$
$= X(1 + Y)$
$= X(1)$
$= X$

c.) $(X + \overline{Y})\, Y = X Y$
$= X Y + \overline{Y} Y$
$= X Y + 0$
$= X Y$

d.) $(X + Y)(\overline{X} + Z) = X Z + \overline{X} Y$
$= X \overline{X} + X Z + \overline{X} Y + Y Z$
$= 0 + X Z + \overline{X} Y + Y Z$
$= X Z + \overline{X} Y + Y Z(1)$
$= X Z + \overline{X} Y + Y Z(X + \overline{X})$
$= X Z + \overline{X} Y + X Y Z + \overline{X} Y Z$
$= (X Z + X Y Z) + (\overline{X} Y + \overline{X} Y Z)$
$= X Z(1 + Y) + \overline{X} Y(1 + Z)$
$= X Z(1) + \overline{X} Y(1)$
$= X Z + \overline{X} Y$

2.9

$XY + YZ + \overline{X}Z$ $\qquad\qquad$ $XY + \overline{X}Z$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$\overline{X}Z$

XY

YZ

2.10

a.)

$f = A(B + CD)$
$\overline{f} = \overline{A(B + CD)}$
$\overline{f} = \overline{A} + \overline{(B + CD)}$
$\overline{f} = \overline{A} + \overline{B} \bullet \overline{(CD)}$
$\overline{f} = \overline{A} + \overline{B}(\overline{C} + \overline{D})$

b.)

$f = ABC + B(\overline{C} + \overline{D})$
$\overline{f} = \overline{ABC + B(\overline{C} + \overline{D})}$
$\overline{f} = \overline{ABC} \bullet \overline{B(\overline{C} + \overline{D})}$
$\overline{f} = (\overline{A} + \overline{B} + \overline{C})(\overline{B} + \overline{(\overline{C} + \overline{D})})$
$\overline{f} = (\overline{A} + \overline{B} + \overline{C})(\overline{B} + CD)$

c.)

$f = \overline{X} + \overline{Y}$
$\overline{f} = X \bullet Y$

d.)

$f = X + Y\overline{Z}$
$\overline{f} = \overline{X + Y\overline{Z}}$
$\overline{f} = \overline{X}\,\overline{(Y\overline{Z})}$
$\overline{f} = \overline{X}(\overline{Y} + Z)$

e.)

$f = (\overline{X} + Y)\overline{Z}$
$\overline{f} = \overline{(\overline{X} + Y)\overline{Z}}$
$\overline{f} = \overline{(\overline{X} + Y)} + Z$
$\overline{f} = X\overline{Y} + Z$

f.)

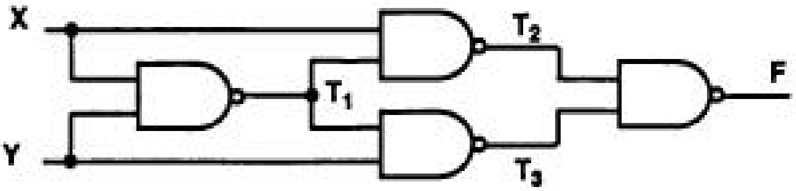
$f = X + \overline{YZ}$
$\overline{f} = \overline{X}(\overline{\overline{YZ}})$
$\overline{f} = \overline{X}(YZ)$

g.)

$f = X(Y + Z\overline{W} + \overline{V}S)$
$\overline{f} = \overline{X(Y + Z\overline{W} + \overline{V}S)}$
$\overline{f} = \overline{X} + \overline{(Y + Z\overline{W} + \overline{V}S)}$
$\overline{f} = \overline{X} + \overline{Y}\,\overline{(Z\overline{W})}\,\overline{(\overline{V}S)}$
$\overline{f} = \overline{X} + \overline{Y}(\overline{Z} + W)(V + \overline{S})$

$\overline{f} = [A + \overline{BCD}][\overline{AD} + B(\overline{C} + A)]$

2.12



$$F = \overline{T_2 \bullet T_3} = \overline{\overline{(X \bullet T_1)} \bullet \overline{(Y \bullet T_1)}} = X\,T_1 + Y\,T_1$$
$$= (X + Y)\,T_1 = (X + Y) \bullet \overline{X\,Y}$$
$$= (X + Y)(\overline{X} + \overline{Y}) = \cancel{X\,\overline{X}} + X\,\overline{Y} + Y\,\overline{X} + \cancel{Y\,\overline{Y}}$$
$$= X\,\overline{Y} + Y\,\overline{X}$$

2.13

a.)

$f(X, Y) = X\,Y + X\,\overline{Y}$	
$= X(Y + \overline{Y})$	Distributive Law
$= X \bullet 1$	Theorem of Complementarity
$= X$	Operations with 0 and 1
One literal	

b.)

$f(X, Y) = (X + Y)\,(X + \overline{Y})$	
$= X\,X + X\,\overline{Y} + Y\,X + Y\,\overline{Y}$	Distributive Law
$= X\,X + X\,\overline{Y} + Y\,X + 0$	Theorem of Complementarity
$= X\,X + X\,\overline{Y} + Y\,X$	Operations with 0 and 1
$= X + X\,\overline{Y} + Y\,X$	Idempotent Theorem
$= X + XY + X\,\overline{Y}$	Commutative Law
$= X$	Simplification Theorem
One literal	

c.)

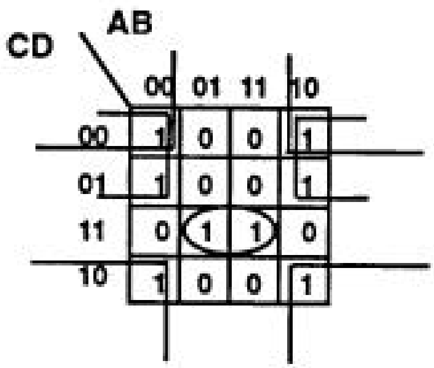
$f(X, Y, Z) = Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,Y\,Z$	
$= Y\,\overline{Z} + (\overline{X} + X)\,Y\,Z$	Distributive Law
$= Y\,\overline{Z} + (1)\,Y\,Z$	Theorem of Complementarity
$= Y\,\overline{Z} + Y\,Z$	Operations with 0 and 1
$= Y(\overline{Z} + Z)$	Distributive Law
$= Y(1)$	Theorem of Complementarity
$= Y$	Operations with 0 and 1
One literal	

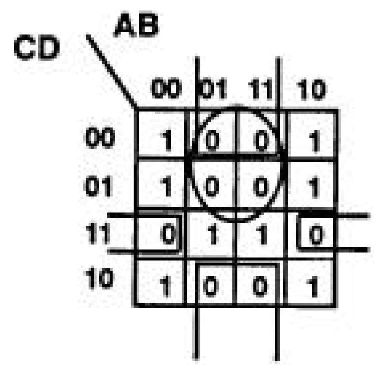
d.)

$f(X, Y, Z) = (X + Y)\,(\overline{X} + Y + Z)\,(\overline{X} + Y + \overline{Z})$	
$= (X + Y)\,(\overline{X} + Y)$	Simplification Theorem
$= Y$	Simplification Theorem
One literal	

2.17

a.) Minimum sum of products form and its complement:



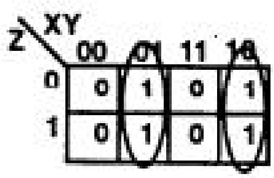


$F = \overline{B}\,\overline{D} + \overline{B}\,\overline{C} + B\,C\,D$

$\overline{F} = B\,\overline{C} + B\,\overline{D} + \overline{B}\,C\,D$

2.18

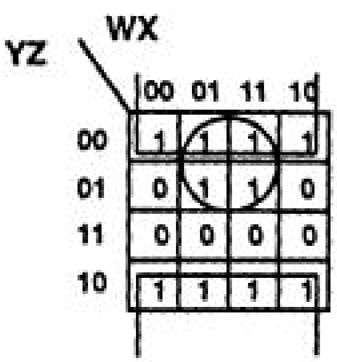
a.)



$f(X, Y, Z) = \overline{X}\,Y + X\,\overline{Y}$

Four literals

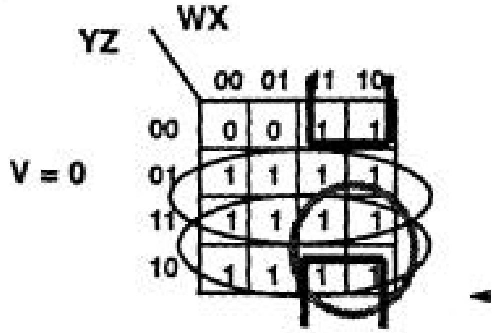
b.)

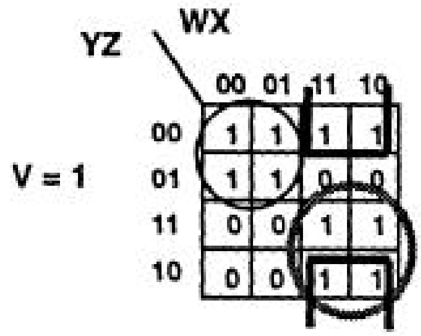


$f(W, X, Y, Z) = \overline{Z} + X\,\overline{Y}$

Three literals

c.)

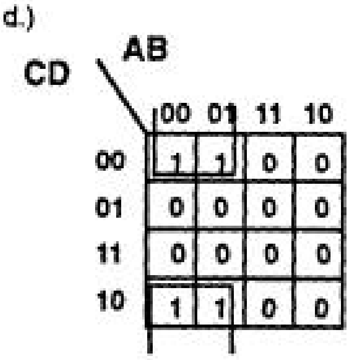




$f(V, W, X, Y, Z) = \overline{V}\,Y + \overline{V}\,Z + W\,\overline{Z} + W\,Y + V\,\overline{W}\,\overline{Y}$

Eleven literals

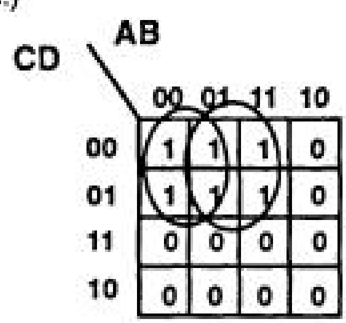
d.)



$f(A, B, C, D) = \overline{A}\,\overline{D}$

Two literals

e.)



$f(A, B, C, D) = \overline{A}\,\overline{C} + B\,\overline{C}$

Four literals

f.)

A = 0

DE \ BC	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

A = 1

DE \ BC	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	1	0	0
10	1	1	0	0

$f(A, B, C, D, E) = A\,\overline{B}\,D + \overline{A}\,\overline{B}\,\overline{D}\,\overline{E} + A\,B\,\overline{D}\,E$

Eleven literals

2.24

CD \ AB	00	01	11	10
00	0	1	1	0
01	1	1	0	0
11	1	0	0	0
10	0	0	0	0

$f(A, B, C, D) = \overline{A}\,\overline{B}\,D + \overline{A}\,B\,\overline{C} + B\,\overline{C}\,\overline{D}$

OR

$f(A, B, C, D) = \overline{A}\,\overline{B}\,D + \overline{A}\,\overline{C}\,D + B\,\overline{C}\,\overline{D}$

2.15

a.) Canonical minterm form:

$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,\overline{D} + A\,B\,C\,D$

b.) Canonical maxterm form:

$\Pi M(3, 4, 5, 6, 11, 12, 13, 14)$

$= (A + B + \overline{C} + \overline{D}) \bullet (A + \overline{B} + C + D) \bullet (A + \overline{B} + C + \overline{D}) \bullet (A + \overline{B} + \overline{C} + D) \bullet (\overline{A} + B + \overline{C} + \overline{D}) \bullet (\overline{A} + \overline{B} + C + D) \bullet (\overline{A} + \overline{B} + C + \overline{D}) \bullet (\overline{A} + \overline{B} + \overline{C} + D)$

c.) Complement of *f* in "little m" notation and as a canonical minterm expression:

$\overline{f} = \Sigma m(3, 4, 5, 6, 11, 12, 13, 14)$

$= \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + \overline{A}\,B\,C\,\overline{D} + A\,\overline{B}\,C\,D + A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,\overline{D}$

d.) Complement of *f* in "big M" notation as a canonical maxterm expression:

$\overline{f} = \Pi M(0, 1, 2, 7, 8, 9, 10, 15)$

$= (A + B + C + D) \bullet (A + B + C + \overline{D}) \bullet (A + B + \overline{C} + D) \bullet (A + \overline{B} + C + \overline{D}) \bullet (\overline{A} + B + C + D) \bullet (\overline{A} + B + C + \overline{D}) \bullet (\overline{A} + B + \overline{C} + D) \bullet (\overline{A} + \overline{B} + \overline{C} + \overline{D})$

2.26

a.)

Σm	A	B	C	D	E	F
0	0	0	0	0	0	0
1	0	0	0	0	1	0
2	0	0	0	1	0	0
3	0	0	0	1	1	1
4	0	0	1	0	0	0
5	0	0	1	0	1	1
6	0	0	1	1	0	1
7	0	0	1	1	1	0
8	0	1	0	0	0	0
9	0	1	0	0	1	1
10	0	1	0	1	0	1
11	0	1	0	1	1	0
12	0	1	1	0	0	1
13	0	1	1	0	1	0
14	0	1	1	1	0	0
15	0	1	1	1	1	0
16	1	0	0	0	0	0
17	1	0	0	0	1	1
18	1	0	0	1	0	1
19	1	0	0	1	1	0
20	1	0	1	0	0	1
21	1	0	1	0	1	0
22	1	0	1	1	0	0
23	1	0	1	1	1	0
24	1	1	0	0	0	1
25	1	1	0	0	1	0
26	1	1	0	1	0	0
27	1	1	0	1	1	0
28	1	1	1	0	0	0

Σm	A	B	C	D	E	F
29	1	1	1	0	1	0
30	1	1	1	1	0	0
31	1	1	1	1	1	0

b.)

F = Σm(3, 5, 6, 9, 10, 12, 17, 18, 20, 24)

c.)

ΠM(0, 1, 2, 4, 7, 8, 11, 13, 14, 15, 17, 19, 21, 22, 23, 25, 26, 27, 28, 29, 30 31)

d.)

A = 0

DE \ BC	00	01	11	10
00	0	0	1	0
01	0	1	0	1
11	1	0	0	0
10	0	1	0	1

A = 1

DE \ BC	00	01	11	10
00	0	1	0	1
01	1	0	0	0
11	0	0	0	0
10	1	0	0	0

$$F = \overline{A}\,\overline{B}\,\overline{C}\,D\,E + \overline{A}\,\overline{B}\,C\,\overline{D}\,E + \overline{A}\,\overline{B}\,C\,D\,\overline{E} + \overline{A}\,B\,C\,\overline{D}\,\overline{E} +$$
$$\overline{A}\,B\,\overline{C}\,\overline{D}\,E + \overline{A}\,B\,\overline{C}\,D\,\overline{E} + A\,\overline{B}\,\overline{C}\,\overline{D}\,E + A\,\overline{B}\,\overline{C}\,D\,\overline{E} +$$
$$A\,\overline{B}\,C\,\overline{D}\,\overline{E} + A\,B\,\overline{C}\,\overline{D}\,\overline{E}$$

2.30

a.)

Σm	D_2	D_1	D_0	C_1	C_0	R_1	R_0
0	0	0	0	0	0	X	X
1	0	0	0	0	1	0	0
2	0	0	0	1	0	0	0
3	0	0	0	1	1	0	0
4	0	0	1	0	0	X	X
5	0	0	1	0	1	0	0
6	0	0	1	1	0	0	1
7	0	0	1	1	1	0	1
8	0	1	0	0	0	X	X
9	0	1	0	0	1	0	0
10	0	1	0	1	0	0	0
11	0	1	0	1	1	1	0
12	0	1	1	0	0	X	X
13	0	1	1	0	1	0	0
14	0	1	1	1	0	0	1
15	0	1	1	1	1	0	1
16	1	0	0	0	0	X	X

Σm	D_2	D_1	D_0	C_1	C_0	R_1	R_0
17	1	0	0	0	1	0	0
18	1	0	0	1	0	0	0
19	1	0	0	1	1	0	1
20	1	0	1	0	0	X	X
21	1	0	1	0	1	0	0
22	1	0	1	1	0	0	1
23	1	0	1	1	1	1	0
24	1	1	0	0	0	X	X
25	1	1	0	0	1	0	0
26	1	1	0	1	0	0	0
27	1	1	0	1	1	0	0
28	1	1	1	0	0	X	X
29	1	1	1	0	1	0	0
30	1	1	1	1	0	0	1
31	1	1	1	1	1	1	0

b.)

$D_2 = 0$

$C_1 C_0$ \ $D_1 D_0$	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	0	0	0	1
10	0	0	0	0

$D_2 = 1$

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	0	1	0	0
10	0	0	0	0

$$R_1 = \overline{D}_2 D_1 D_0 C_1 C_0 + D_2 \overline{D}_1 D_0 C_1 C_0$$

$D_2 = 0$

$C_1 C_0$ \ $D_1 D_0$	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	0	1	0	0
10	0	1	1	0

$D_2 = 1$

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	1	0	1	0
10	0	1	1	0

$$R_0 = D_0 \overline{C}_0 + \overline{D}_2 \overline{D}_1 D_0 C_1 + D_2 D_1 D_0 C_1$$
$$+ D_2 \overline{D}_1 \overline{D}_0 C_1 C_0$$