Professor Fearing **Fall 1998**

CS 150 Problem Set 11

Solutions by Nadeem (nma@cory.eecs)

1. K-Maps

Here is the truth table for the function:

$A_3A_2A_1A_0$	$\log_2$	BC.DE
0000	x	xx.xx
0001	0	00.00
0010	1	01.00
0011	1.58	01.10
0100	2	10.00
0101	2.32	10.01
0110	2.59	10.10
0111	2.81	10.11
1000	3	11.00
1001	3.17	11.01
1010	3.32	11.01
1011	3.46	11.10
1100	3.58	11.10
1101	3.70	11.11
1110	3.81	11.11
1111	3.91	11.11

B is 1 when the input is 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15, while 0 is a don't care value. This is written as:

$B = \Sigma(4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15) + D(0)$

Now fill in the K-Map table with 0, 1, and X and circle all of the entries where B is 1.

A_3A_2 \ A_1A_0	00	10	11	10	
00	x	0	0	0	
01	1	1	1	1	A_2
11	1	1	1	1	
10	1	1	1	1	A_3

Next you can figure out the boolean equation by looking at which values are circled. There will be one sum term for each circle. In this case, one circle corresponds to when A_2 is 1 and the other to when A_3 is 1. So, the equation is:

$$B = A_2 + A_3$$

$C = \Sigma(2, 3, 8, 9, 10, 11, 12, 13, 14, 15) + D(0)$

A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	1	1
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

$C = A_3 + A_1\overline{A_2}$

$D = \Sigma(3, 6, 7, 11, 12, 13, 14, 15) + D(0)$

A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	1	0
01	0	0	1	1
11	1	1	1	1
10	0	0	1	0

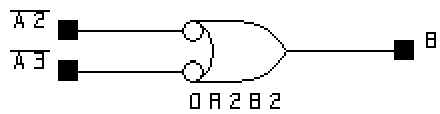
$D = A_1A_0 + A_3A_2 + A_2A_0$

$E = \Sigma(5, 7, 11, 12, 13, 14, 15) + D(0)$

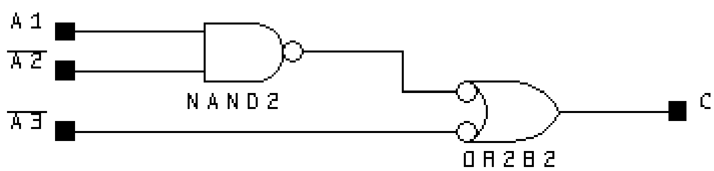
A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	0	0
01	0	1	1	0
11	0	1	1	1
10	0	1	0	1

$E = A_0A_2 + A_3\overline{A_1}A_0 + A_3A_1\overline{A_0}$

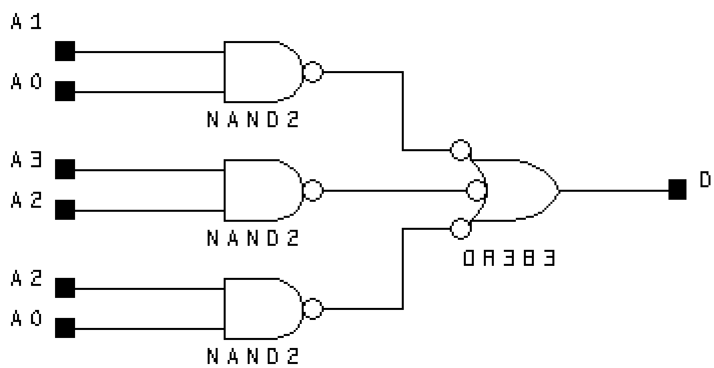
$B = A_2 + A_3$



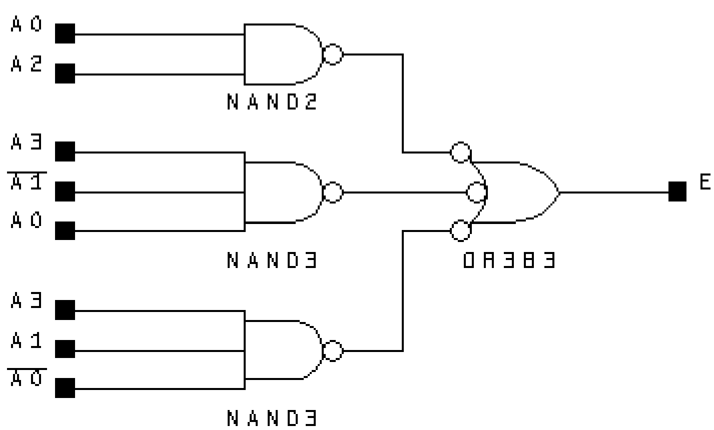
$C = A_3 + A_1\overline{A_2}$



$D = A_1A_0 + A_3A_2 + A_2A_0$



$E = A_0A_2 + A_3\overline{A_1}A_0 + A_3A_1\overline{A_0}$



To find Product of Sums form, cirlce the 0s and invert the equation.

$\overline{B} = \Sigma(1, 2, 3) + D(0)$

A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	0	0
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\overline{B} = \overline{A_3}\,\overline{A_2}$

$B = \overline{\overline{A_3}\,\overline{A_2}}$

$B = A_2 + A_3$

$\overline{C} = \Sigma(4, 5, 6, 7) + D(0)$

A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	1	1
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

$C = (A_3 + A_1)(A_3 + A_1)$

$\overline{D} = \Sigma(1, 2, 4, 5, 8, 9, 10) + D(0)$

A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	1	0
01	0	0	1	1
11	1	1	1	1
10	0	0	1	0

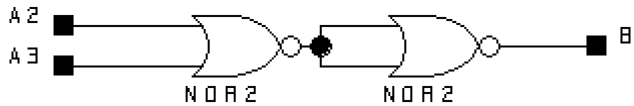
$D = (A_3 + A_1)(A_2 + A_1)(A_0 + A_2)$

$\overline{E} = \Sigma(1, 2, 3, 4, 6, 8, 11, 12) + D(0)$

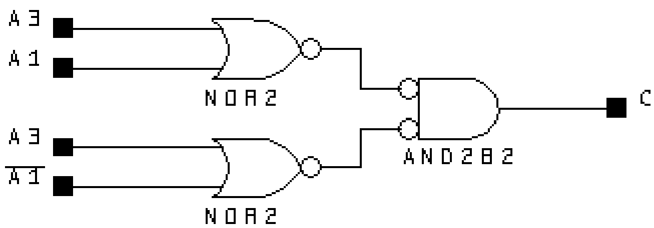
A_3A_2 \ A_1A_0	00	10	11	10
00	x	0	0	0
01	0	1	1	0
11	0	1	1	1
10	0	1	0	1

$E = (\overline{A_3} + \overline{A_2})(\overline{A_1} + \overline{A_0})(\overline{A_0} + \overline{A_3})(A_2 + \overline{A_1} + \overline{A_0})$

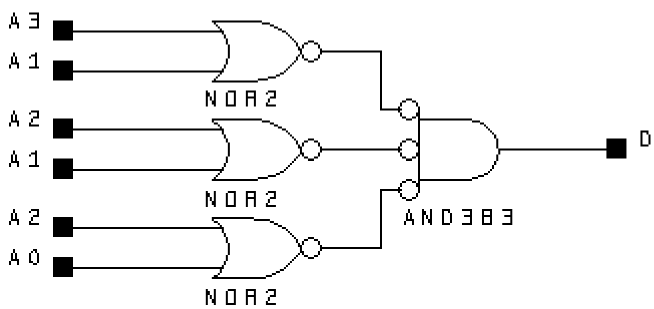
$B = A_2 + A_3$



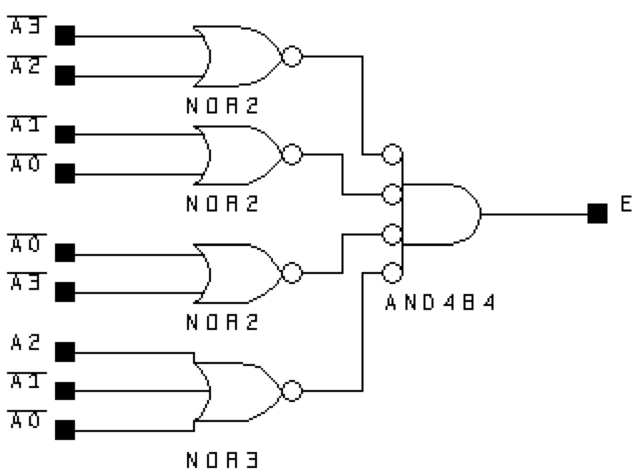
$C = (A_3 + A_1)(A_3 + \overline{A_1})$



$D = (A_3 + A_1)(A_2 + A_1)(A_0 + A_2)$



$E = (\overline{A_3} + \overline{A_2})(\overline{A_1} + \overline{A_0})(\overline{A_0} + \overline{A_3})(A_2 + \overline{A_1} + \overline{A_0})$



2. State Minimization

Present State	Next State		Output
	IN = 0	IN = 1	
A	A	C	0
B	D	B	1
C	D	B	0
D	B	C	0
E	E	F	0
F	G	B	0
G	B	F	0

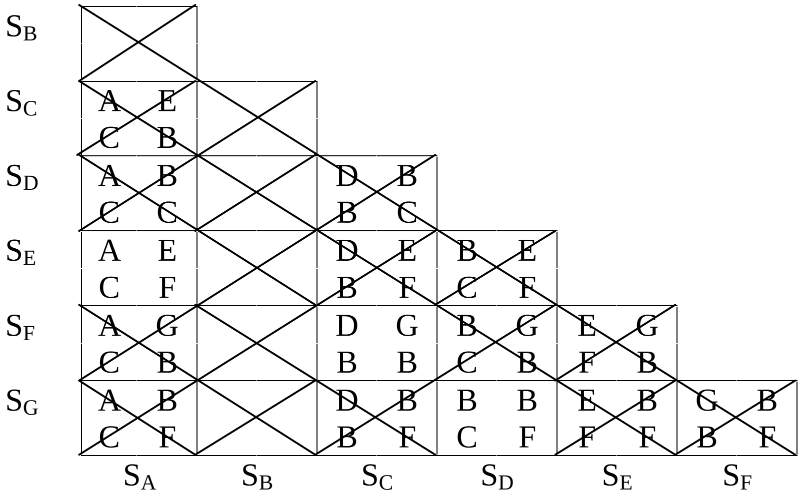
Fill out every entry in the state table with the next states for each input. The following is the first entry for states A and B:

S_A	S_B	
A	D	IN = 0
C	B	IN = 1

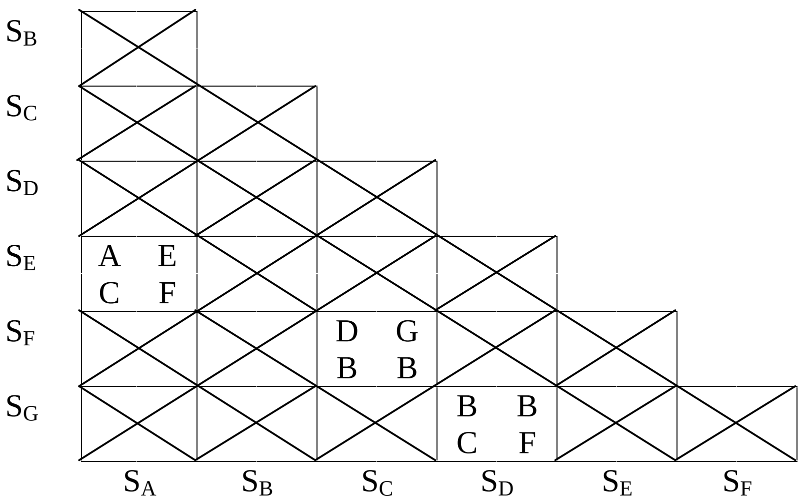
After filling out the table, first cross out the pairs that have different outputs. In this case, state B is the only state that outputs a 1, so cross out the states paired with state B.

S_B	~~A D~~ ~~C B~~					
S_C	A E C B	~~D D~~ ~~B B~~				
S_D	A B C C	~~D B~~ ~~B C~~	D B B C			
S_E	A E C F	~~D E~~ ~~B F~~	D E B F	B E C F		
S_F	A G C B	~~D G~~ ~~B B~~	D G B B	B G C B	E G F B	
S_G	A B C F	~~D B~~ ~~B F~~	D B B F	B B C F	E B F F	G B B F
	S_A	S_B	S_C	S_D	S_E	S_F

Next, look for the state pairs within each cell that have already been crossed out. This means the states are not equivalent, so cross it out. In this case, any state paired up state B as the next state should be crossed out.



Check again if there any more state pairs that have next states that are not equivalent.



Equivalent states: E = A, C = F, D = G.